



**QUEEN'S
UNIVERSITY
BELFAST**

Dissertation: Conditional Variance Modeling of Financial Time Series: Essays on Advances and Applications to Commodity and Foreign Exchange Markets

Klein, T. (2017, Dec 8). Dissertation: Conditional Variance Modeling of Financial Time Series: Essays on Advances and Applications to Commodity and Foreign Exchange Markets. Unknown Publisher.

Document Version:
Peer reviewed version

Queen's University Belfast - Research Portal:
[Link to publication record in Queen's University Belfast Research Portal](#)

Publisher rights
Copyright 2017 The Author.

General rights
Copyright for the publications made accessible via the Queen's University Belfast Research Portal is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy
The Research Portal is Queen's institutional repository that provides access to Queen's research output. Every effort has been made to ensure that content in the Research Portal does not infringe any person's rights, or applicable UK laws. If you discover content in the Research Portal that you believe breaches copyright or violates any law, please contact openaccess@qub.ac.uk.

Conditional Variance Modeling of Financial Time Series

Essays on Advances and Applications to Commodity and
Foreign Exchange Markets

Doctoral Dissertation
in partial fulfillment of the requirements for the degree
Dr. rer. pol.

Author

Dipl.-Math. Tony Klein

born October 6, 1987 in Dresden, Germany

First Supervisor

Prof. Dr. Hermann Locarek-Junge

Chair of Finance and Financial Services

Second Supervisor

Prof. Dr. Bernhard Schipp

Chair of Quantitative Methods, esp. Econometrics

Faculty of Business and Economics

Technische Universität Dresden

Submission: May 31, 2017

Defense: December 8, 2017

Acknowledgements

First and foremost, I would like to thank my first supervisor, Professor Hermann Locarek-Junge, for his trust and valuable support during my research. Each conference I was able to attend and every presentation I could give lead to an improvement of this present thesis. I appreciate the comments and advice of Professor Bernhard Schipp, my second supervisor, during the final stage of my doctoral studies.

I am especially grateful to Professor Dirk Linowski for the intellectual support, the steady motivation, and the many fruitful and pleasant discussions of academic and personal nature.

My thanks go to Thomas Walther, whom I started this academic adventure with. He not only became a colleague and co-author, but also a valued friend of mine.

I am thankful to Professor David Kemme and Professor William T. Smith for inviting me to Memphis, TN, to be a visiting scholar at The University of Memphis, Fogelman College of Business & Economics, whose hospitality and excellent research environment are greatly appreciated.

The financial support of the Graduate Academy, Technische Universität Dresden, financed by *The Excellence Initiative* of the German Federal Ministry of Education and Research (BMBF) and the German Research Foundation (DFG), for my visit in Memphis is gratefully acknowledged. I am equally thankful for receiving a travel grant from the Deutsche Bundesbank supporting my visit in Memphis.

I am thankful for the support and the productive collaboration with the dean's office, particularly Professor Michael Schefczyk, Professor Udo Buscher, and Dr. Uta Schwarz, throughout my time at the Faculty of Business and Economics.

Lastly, I want to thank all those unnamed who supported me directly or indirectly with reliable advice, constructive criticism, or a friendly ear during my academic adventures of the last three and a half years as a doctoral candidate in Dresden.

Contents

| | |
|---|-------------|
| List of Figures | VI |
| List of Tables | VII |
| Abbreviations | VIII |
| Summary of Notations | IX |
| 1 Introduction | 1 |
| 2 Research Articles | 5 |
| 2.1 Oil Price Volatility Forecast with Mixture-Memory-GARCH Models . . | 5 |
| 2.2 Fast Fractional Differencing in Modeling Long Memory of Conditional Variance for High-Frequency Data | 6 |
| 2.3 Empirical Evidence of Long Memory and Asymmetry in EUR/PLN Exchange Rate Volatility | 7 |
| 2.4 True or Spurious Long Memory in European Non-EMU Currencies . . . | 7 |
| 2.5 The Choice of Realized Volatility Measures for Forecast Evaluation - An Application to Gold and Silver | 8 |
| 2.6 Dynamic Correlation of Precious Metals and Flight-to-Quality in De- veloped Markets | 9 |
| 2.7 Trend Contagion in WTI and Brent Crude Oil Spot and Futures Prices - A Spread and Correlation Analysis | 10 |
| 2.8 Other Articles | 10 |
| 2.9 Topical Relatedness | 11 |
| 3 Selected Topics | 13 |
| 3.1 Application of Fast Fractional Differencing | 13 |
| 3.1.1 Methodological Motivation | 13 |
| 3.1.2 Convolution Sum Timing | 17 |
| 3.1.3 Numerical Precision of Different Calculation Schemes | 19 |
| 3.1.4 Application in Rolling-Window Variance Forecasting | 20 |
| 3.2 Properties of Commodity Spot and Futures Returns | 23 |
| 3.2.1 Precious Metals—On Assets and Industrial Metals | 23 |
| 3.2.2 Similarities to Crude Oil Returns and Concluding Remarks . . . | 27 |
| 4 Conclusions and Outlook | 29 |
| References | 31 |

List of Figures

| | | |
|---|--|----|
| 1 | Interconnectedness of all published articles and working papers. | 12 |
| 2 | Computation time for different implementations and sample lengths. . . | 18 |
| 3 | Mean difference of σ_{VS}^2 and σ_{FFT}^2 under the L_2 and L_∞ norm. | 20 |
| 4 | Total times of out-of-sample FIGARCH parameter estimations (WTI). . . | 21 |
| 5 | Parameter estimation times of FIAPARCH for single estimation. | 22 |
| 6 | Gold and Silver prices and variance measures before and after the publication of the Brexit poll results on 24-Jun-2016. | 27 |

List of Tables

| | | |
|---|---|----|
| 1 | Mean computation time for FIGARCH with $n = 15\,000$ | 17 |
| 2 | Speed-ups of FFT for a sample length $n = 15\,000$ | 18 |
| 3 | Speed-ups of FFT compared to VS for an analysis of FIGARCH. . . . | 21 |
| 4 | Speed-ups of FFT for FIAPARCH parameter estimation. | 22 |
| 5 | Parameter estimates for Gold spot and futures returns between 01-Jan-2000 and 31-Dec-2016. | 24 |
| 6 | Parameter estimates for Silver spot and futures returns between 01-Jan-2000 and 31-Dec-2016. | 25 |
| 7 | Parameter estimates for Platinum and Palladium spot returns between 01-Jan-2000 and 31-Dec-2016. | 26 |
| 8 | Asymmetric and long memory properties of WTI and Brent spot and selected futures prices from 01-Jan-2007 to 29-Jul-2016 ($n = 2\,421$). . . | 28 |

Abbreviations

General

| | |
|---------|---|
| BIC | Bayesian Information Criterion |
| CDT/CST | Central Daylight Time/Central Standard Time |
| DGP | Data Generating Process |
| EIA | U.S. Energy Information Administration |
| EM | Expectation-Maximization |
| EMU | European Monetary Union |
| FFT | Fast Fourier Transforms |
| LL | Log-likelihood |
| i.i.d. | Independent and Identically Distributed |
| ItS | Iterative Summation |
| MCS | Model Confidence Set |
| OPEC | Organization of the Petroleum Exporting Countries |
| RMSE | Root Mean Squared Error |
| SPA | Superior Predictive Ability |
| VaR | Value-at-Risk |
| VS | Vectorized Summation |
| WTI | West Texas Intermediate |

Model Abbreviations

| | |
|-------------------|---|
| AR | Autoregressive |
| —MA | Autoregressive Moving Average |
| —FIMA | Autoregressive Fractionally Integrated Moving Average |
| APARCH | Asymmetric Power ARCH |
| ARCH | Autoregressive Conditional Heteroskedasticity |
| BEKK | BABA, ENGLE, KRAFT, KRONER model |
| CAPM | Capital Asset Pricing Model |
| DCC | Dynamic Conditional Correlation |
| EGARCH | Exponential GARCH |
| FIAPARCH | Fractionally Integrated Asymmetric Power ARCH |
| FIGARCH | Fractionally Integrated GARCH |
| GARCH | Generalized ARCH |
| GARCH- <i>vec</i> | Multivariate vectorized GARCH |
| HYGARCH | Hyperbolic GARCH |
| MMGARCH | Mixture Memory GARCH |
| VAR | Vector Autoregressive |

Summary of Notations

Indexes and Number Sets

| | |
|-------------------------------|--|
| t | Index for observation times, $t = 1, \dots, n$ |
| n, n_{sample} | Sample length |
| n_{trunc} | Truncation lag |
| N | Number of repetitions of an experiment |
| $K.$ | Number of out-of-sample points with specifier as index |
| \mathbb{N} | Set of natural numbers |
| $\mathbb{R}, \mathbb{R}_{>0}$ | Set of real numbers, positive real numbers |
| \mathbb{Z} | Set of integer numbers |

Parameters

| | |
|-----------------------|---|
| α, β, ϕ | Parameters of the respective variance model |
| γ | Leverage parameter |
| δ | Box-Cox power transformation parameter |
| d | Fractional differencing parameter |
| λ_i | Fractional differencing weights, $i = 1, \dots, n_{\text{trunc}}$ |
| ω | Variance intercept |

Distributions and Distribution Properties

| | |
|------------------------------|--|
| $\mathcal{N}(\mu, \sigma^2)$ | Normal or Gaussian Distribution with mean $\mu \in \mathbb{R}$ and variance σ^2 with $\sigma \in \mathbb{R}_{>0}$ |
| $\mathcal{N}(0, 1)$ | Standard Normal Distribution |
| $\text{St-}t_\nu(0, 1)$ | Standardized Student's- t Distribution with $\nu > 2$ degrees of freedom, zero mean, and variance one |
| σ, σ^2 | Unconditional volatility, unconditional variance |
| σ_t^2 | Conditional variance at time t |
| μ_t | Conditional mean at time t |

Other

| | |
|-------------------|--|
| L | Lag operator with $L^m x_t = x_{t-m}$ for $m \in \mathbb{N}$ and $t - m > 0$ |
| \mathcal{F}_t | 'Information set', σ -algebra generated by the past of the model framework up to time t |
| $\{a_t\}_{t=1}^n$ | Finite sequence with elements a_1, a_2, \dots, a_n |

1 Introduction

“Our word is a modern coinage, derived from the two Greek roots hetero— ($\epsilon\tau\epsilon\rho\alpha$), meaning ‘other’ or ‘different,’ and skedannumi ($\sigma\kappa\epsilon\delta\acute{\alpha}\nu\nu\mu\iota$), meaning ‘to scatter.’”
McCulloch (1985, p. 483)

Heteroskedasticity¹—meaning to be of different dispersion—is a fundamental concept in statistics and applied econometrics. Using the variance, if existing, as measure of dispersion, heteroskedasticity translates to varying variance over subsamples or -periods. In time series analysis, this heteroskedastic variance is then said to be time-varying, leading to phenomena like volatility clustering and different degree of decay of its high levels. Simple examples are times of market turmoil, such as the 1973 oil crisis or longer periods like the financial crisis of 2008 and its transmission to the European debt crisis. These periods are characterized by massive price movements preceded and followed by calmer trading periods, forming pronounced volatility clusters. In the pioneer work of Engle (1982), this auto-correlated and time-varying variance is formalized and modeled in the *Autoregressive Conditional Heteroskedasticity* (ARCH) model. This groundbreaking advance paves the way for a wide class of variance models with a variety of financial and non-financial applications.

In addition to clustering, the variance of financial return series features additional properties such as a slow or fast decay of shocks, asymmetry, or their combinations. These properties are of asset-specific magnitude which is further promoted in this thesis. An overview of some of these so-called ‘stylized facts’ is found in Cont (2001).

Several extensions of the ARCH model of Engle (1982) are constructed to account for a variety of properties and implemented throughout the essays of this thesis. The simplest form and often included as a sparsely parameterized benchmark is the *Generalized ARCH* (GARCH) of Bollerslev (1986). For distinguishing between the impact of positive and negative returns on the variance—closely related to ‘good’ and ‘bad’ news or events—the *Asymmetric Power ARCH* (APARCH) of Ding et al. (1993) is used. Covering long memory (slowly declining shocks), the *Fractionally Integrated ARCH* (FIGARCH) of Baillie et al. (1996) is included. The FIGARCH utilizes fractional differencing to depict a hyperbolically decaying persistence in squared returns. Unifying the concept of long memory and asymmetry in a single model, the *Fractionally Integrated Asymmetric Power ARCH* (FIAPARCH) of Tse (1998) is applied. Several other processes covering similar properties such as the *Exponential GARCH*

¹We adapt the orthography clarified in McCulloch (1985).

(EGARCH, Engle & Ng, 1993) or *Hyperbolic* GARCH (HYGARCH, Davidson, 2004) are compared in Klein & Walther (2016).

Shortly after its univariate definition, the ARCH/GARCH framework is extended to a multivariate setting in Bollerslev et al. (1988) and Engle & Kroner (1995), which is carried out in analogy to Vector Autoregressive (VAR) processes (Sims, 1980). This is of particular interest to financial applications and motivated by variance modeling of a portfolio of several assets. Within the *Capital Asset Pricing Model* (CAPM, Sharpe, 1964; Lintner, 1965; Mossin, 1966) which is based on the pioneer work of Markowitz (1952), the expected return of an asset is dependent on the covariance with the market and its volatility. Given a multivariate variance model, Bollerslev et al. (1988) now extend the CAPM to time varying variance and correlation.

For the multivariate modeling of variance-covariance matrices, different approaches exist as parameter parsimony is at stake. A simple, two-dimensional GARCH(1,1)-*vec* model, where *vec* refers to stacking of the matrices, already requires nine parameters.² The BEKK model³ outlined in Engle & Kroner (1995, p. 5) introduces a feasible solution to this issue. With the focus on correlation modeling, Engle (2002) introduces the *Dynamic Conditional Correlation* (DCC) model that simplifies the multivariate GARCH framework. Other alternatives are presented in Tse & Tsui (2002), for example. The DCC is very flexible and relatively easy to modify (see further: Mensi et al., 2017; Klein, 2017), but not free of criticism (Caporin & McAleer, 2013); for example possible inconsistencies in the multiple-step estimators.

Modeling the time-varying variance and covariance with distinct, asset-specific features has several practical applications. In the univariate case, forecasting or predicting future variances is conducted by calculating their expected value based on daily observations. We make use of these predictions to forecast the Value-at-Risk (VaR) for up to 20 days ahead. The VaR and its violations are evaluated for *short* and *long* positions in an asset or security. A long position refers to holding an asset (e.g. stocks), the obligation to buy at a future time (e.g. futures contracts), or the right to buy at a future time (e.g. options) to name a few examples. Then, the respective counter-party of the trade is in short position.

The quality of predicted variances is evaluated with so-called loss functions. An example is the root mean squared error (RMSE).⁴ With bootstrapping methods, the loss functions are then analyzed to identify the preferred variance model. Hansen's *Superior Predictive Ability* (SPA, Hansen, 2005) or the more sophisticated *Model*

²In a k -dimensional model, $\frac{1}{4}(k(k+1))^2$ unique parameters need to be estimated (Engle & Kroner, 1995, pp. 2f.).

³An earlier version of this model was formulated by Yoshi Baba, Robert F. Engle, Dennis Kraft, and Kenneth Kroner.

⁴More choices are introduced and applied in Klein & Walther (2016, p. 50).

Confidence Set (MCS, Hansen et al., 2011) are the standard choices to carry out this task. For an overview on the VaR and its tests, it is referred to Piontek (2010).

In the multivariate setting, the applications of variance models are broader given the simultaneous modeling of variances and covariances. Time-varying correlation and contagion effects between markets or assets (e.g. Celik, 2012; Sensoy, 2013; Chkili, 2016) and co-movements can be identified and modeled within the respective frameworks. Hedging strategies or asset-specific behavior such as a safe-haven status (Baur & Lucey, 2010) are of importance to portfolio management and diversification as well as a general understanding of global market dynamics.

In this thesis, two different classes of research objects are focused on. Firstly, the relatively broad field of commodities is narrowed down by examining the properties of crude oil blend prices such as the North-American *West Texas Intermediate* (WTI), the European *Brent*, and the basket price of the Organization of the Petroleum Exporting Countries (OPEC). Precious metal prices are analyzed with regard to their behavior in times of market turmoil—based on their very specific and unique econometric properties. On daily basis, Gold and Silver are found to react asymmetrically to shocks (Arouri et al., 2012, p. 212f.; Chkili et al., 2014, p. 17) while in the long run, correlations to indexes or market sectors are of high interest in portfolio diversification (Hammoudeh et al., 2010, p. 633; Baur & Lucey, 2010, pp. 217f.). Other precious metals with a more pronounced industrial application differ in their characteristics. Secondly, foreign exchange rates are examined in regard to their variance behavior. Similar to the applications named above, hedging exchange rate risks is of utmost importance for companies and banks which have international relations.

The scholarly advancements presented in the essays of this thesis are versatile in application but closely related with regard to variance modeling. The understanding of varying shock persistence in variance and trends in prices and returns of crude oil is improved by applying new models. Market mechanisms and spillover effects are identified and support or extend the current literature. The finding that OPEC meetings and their decisions have only little effect on long-term trends in crude oil prices in the last decade is novel and motivates further research.

Introducing fast Fourier transforms (FFTs) to long memory models has a large impact on the research presented herein. All long memory models or mixtures thereof (e.g. *Mixture-Memory-GARCH* in Li et al. (2013) and application in Klein & Walther (2016)) benefit tremendously by lowering calculation times which makes the application of long memory models on extended data ranges (such as intra-day data) feasible. This novel modification greatly amplifies the applicability of these models.

For the examined exchange rates, it is proven that long memory plays a vital role

in variance and has to be included in risk management applications. The detection of so-called spurious long memory is advanced by proposing new model variants. These models had not yet been applied on non-*European Monetary Union* (EMU) currency pairs.

With the introduction of a variable approach to econometric properties in the DCC model, correlation depiction is improved as asset-specific features are included. Modeling the correlation of precious metals and different indexes, the findings obtained with this adjusted model support recent literature and present new evidence of safe-haven surrogates in extreme market situations.

The following articles are part of this dissertation and described in detail in Chapter 2. In this thesis, Papers 1-7 are referenced as such if the linkage between these essays is of interest. Otherwise the name-year reference is used.

Paper 1 (MM):

Oil Price Volatility Forecast with Mixture-Memory-GARCH Models

Paper 2 (FF):

Fast Fractional Differencing in Modeling Long Memory of Conditional Variance for High-Frequency Data

Paper 3 (FX):

Empirical Evidence of Long Memory and Asymmetry in EUR/PLN Exchange Rate Volatility

Paper 4 (LM):

True or Spurious Long Memory in European Non-EMU Currencies

Paper 5 (PM):

The Choice of Realized Volatility Measures for Forecast Evaluation - An Application to Gold and Silver

Paper 6 (PM-Corr):

Dynamic Correlation of Precious Metals and Flight-to-Quality in Developed Markets

Paper 7 (Trends):

Trend Contagion in WTI and Brent Crude Oil Spot and Futures Prices - A Spread and Correlation Analysis

The remainder of this dissertation is as follows. Chapter 2 provides an overview of seven research articles in form of abstracts and a concluding depiction of their topical relatedness. Chapter 3 offers additional insight on some of the papers and presents supplementary results. This dissertation concludes with Chapter 4 including an outlook for further research.

2 Research Articles

This chapter introduces the original research articles that compile the cumulative dissertation in the following sections. The abstracts are taken from the publication or working paper. A list of conferences and seminars, where the respective paper was presented and discussed, concludes each article section. If the author of this dissertation has not been the presenter, the presenting co-author is given in parentheses.

Since all articles are based on each other or otherwise directly related, a detailed representation of these linkages is formulated in Section 2.9, which concludes this chapter. The following order is roughly based on when the work on the respective paper had started. Each full paper is attached in the Appendix in the order adopted in this section. *All articles are attached in their native, journal-specific format, citation style, and pagination.*

2.1 Oil Price Volatility Forecast with Mixture-Memory-GARCH Models

Referenced as: Paper 1 (MM) and Klein & Walther (2016)

Abstract

We expand the literature of volatility and Value-at-Risk forecasting of oil price returns by comparing the recently proposed Mixture-Memory-GARCH (MMGARCH) model to other discrete volatility models (GARCH, RiskMetrics, EGARCH, APARCH, FIGARCH, HYGARCH, and FIAPARCH). We incorporate an Expectation-Maximization (EM) algorithm for parameter estimation of the MMGARCH and find different structures in volatility level as well as shock persistence. MMGARCH is also able to cover asymmetric and long memory effects. Furthermore, a dissimilar memory structure in variance of WTI and Brent crude oil prices is observed which is supported by additional tests. Parameter estimation and comparison of the models reveal significant long memory and asymmetry in oil price returns. In regard of variance forecasting and Value-at-Risk prediction, it is shown that MMGARCH outperforms the aforementioned models due to its dynamic approach in varying the volatility level and memory of the process. We find MMGARCH superior for application in risk management as a result of its flexibility in adjusting to variance shifts and shocks.

Published as:

Klein, Tony; Walther, Thomas (2016): Oil Price Volatility Forecast with Mixture-Memory-GARCH Models, in: Energy Economics, Vol. 58, pp. 46-58, DOI: 10.1016/j.eneco.2016.06.004.

Presented and discussed at:

- International Ruhr Energy Conference (INREC) 2015, Essen, Germany
- Energy Finance Conference 2015, London, UK
- HypoVereinsBank PhD Seminar Fall 2015, Riga, Latvia

2.2 Fast Fractional Differencing in Modeling Long Memory of Conditional Variance for High-Frequency Data

Referenced as: Paper 2 (FF) and Klein & Walther (2017)

Abstract

We transfer the recently introduced fast fractional differencing that utilizes fast Fourier transforms to long memory variance models and show that this approach offers immense computation speedups. We demonstrate how calculation times of parameter estimations benefit from this new approach without changing the estimation procedure. A more precise depiction of long memory behavior becomes feasible. The FFT offers a computational advantage to all ARCH(∞)-representations of widely-used long memory models like FIGARCH. Risk management applications like rolling-window Value-at-Risk predictions are substantially sped up. This new approach allows to calculate the conditional volatility of high-frequency in a practicable amount of time.

Published as:

Klein, Tony; Walther, Thomas (2017): Fast Fractional Differencing in Modeling Long Memory of Conditional Variance for High-Frequency Data, in: Finance Research Letters, Vol. 22C, pp. 274-279, DOI: 10.1016/j.frl.2016.12.020.

Presented and discussed at:

- Workshop of the German Operations Research Society (GOR e.V.), WG FIFL, April 2016, Augsburg, Germany
- HypoVereinsBank PhD Seminar Spring 2016, Halle/S., Germany
- HSC Seminar on Stochastic and Numerical Methods, 2016, Wrocław University of Technology, Wrocław, Poland
- Vietnam International Conference in Finance 2016, Da Nang, Vietnam (by Co-Author Thomas Walther)
- Macromodels International Conference, Łódź, Poland, 2016 (by Co-Author Thomas Walther)

2.3 Empirical Evidence of Long Memory and Asymmetry in EUR/PLN Exchange Rate Volatility

Referenced as: Paper 3 (FX) and Klein et al. (2016)

Abstract

This paper focuses on forecasting the conditional volatility of foreign exchanges. By implementing a variety of GARCH models, we examine the volatility of daily returns of EUR/PLN exchange rates. A significant long memory and asymmetry effect in volatility are confirmed. These characteristics implicate some challenges in volatility forecasting and Value-at-Risk predictions in short and long trading positions. Therefore, we combine these two effects in joint modeling framework which yields the best goodness-of-fit of all aforementioned models. Furthermore, it outperforms other models in regard to the applied loss functions and is found to provide the best Value-at-Risk estimation results. Our findings contribute to research on volatility of Polish exchange rate, expand the findings related to dynamic volatility in the existing literature and raise awareness of combined volatility effects to practitioners.

Published as:

Klein, Tony; Pham Thu, Hien; Walther, Thomas (2016): Empirical Evidence of Long Memory and Asymmetry in EUR/PLN Exchange Rate Volatility, in: Research Papers of Wroclaw University of Economics, No. 428, pp. 128-140, DOI: 10.15611/pn.2016.428.11.

Presented and discussed at:

- Wroclaw Conference in Finance (WROFIN) 2015, Wrocław, Poland (by Co-Author Thomas Walther)
- Science meets Social Science (S3) Seminar 2015, Wrocław University of Technology, Poland (by Co-Author Thomas Walther)

2.4 True or Spurious Long Memory in European Non-EMU Currencies

Referenced as: Paper 4 (LM) and Walther et al. (2017)

Abstract

We examine the Croatian Kuna, the Czech Koruna, the Hungarian Forint, the Polish Złoty, the Romanian Leu, and the Swedish Krona whether their Euro exchange rate's volatility exhibits true or spurious long memory. Recent research reveals long memory in foreign exchange rate volatility and we confirm this finding for these currency pairs by examining the long memory behavior of squared residuals by means of the V/S

test. However, by using the ICSS approach we also find structural breaks in the unconditional variance. Literature suggests that structural breaks might lead to spurious long memory behavior. In a refined test strategy, we distinguish true from spurious long memory for the six exchange rates. Our findings suggest that Czech Koruna and Hungarian Forint only feature spurious long memory, while the rest of the series have both structural breaks and true long memory. Lastly, we demonstrate how to extend existing models to jointly model both properties yielding superior fit and better Value-at-Risk forecasts. The results of our work help to avoid misspecification and provide a better understanding of the properties of the foreign exchange rate volatility.

Published as:

Walther, Thomas; Klein, Tony; Pham Thu, Hien; Piontek, Krzysztof (2017): True or Spurious Long Memory in European Non-EMU Currencies, in: Research in International Business and Finance, Vol. 40, pp. 217-230, DOI: 10.1016/j.ribaf.2017.01.003.

Presented and discussed at:

- Macromodels International Conference, Łódź, Poland, 2016
- Wrocław Conference in Finance (WROFIN) 2016, Wrocław, Poland (by Co-Author Thomas Walther)

Awards:

- AMFET Best Paper Award, Macromodels International Conference, 2016
- Certificate of Appreciation, Wrocław Conference in Finance, 2016

2.5 The Choice of Realized Volatility Measures for Forecast Evaluation - An Application to Gold and Silver

Referenced as: Paper 5 (PM); in revision (round 2)

Abstract

The conditional variance of returns of Gold and Silver is modeled incorporating GARCH-type processes. Gold and Silver are dominated by asymmetric effects which is verified by taking into account recent shocks. The forecasting comparison of these models over varying horizons is compared with two different proxies for the daily realized variance—squared daily returns, which is the standard choice in literature, and a proxy based on intra-day data. It is found that incorporating a more realistic measure for the realized volatility, the loss functions are significantly reduced and the decision on the best performing model changes. For our data set of Gold and Silver returns, applying the standard proxy leads to a wrong model decision of superior variance

prediction. For Gold and Silver, APARCH and partly FIAPARCH are found to be the best performing in goodness-of-fit and variance prediction.

An earlier version was presented and discussed at:

- Economics Seminar, Fogelman College of Business and Economics, University of Memphis, 2016

2.6 Dynamic Correlation of Precious Metals and Flight-to-Quality in Developed Markets

Referenced as: Paper 6 (PM-Corr) and Klein (2017)

Abstract

A flexible modification of the DCC model that accounts for asymmetry and long memory in variance is proposed. An outline of an iterative algorithm that estimates variance models and the DCC with a heavy tailed distribution is given. This model is applied on precious metals and indexes of developed countries to revisit the flight-to-quality phenomenon. Market turmoil and high index volatilities are covered by asset-specific variance models which allow the modeling of shocks and asymmetric news impact; important factors which should not be neglected in dynamic correlation modeling. Gold and, in parts, Silver are found to serve as safe haven while this status seems to be dissipating in the recent years. Interestingly, Platinum shows signs of a surrogate safe haven. The practical difference between the standard DCC and the model proposed herein is significant, especially in periods of market reversals and shocks which stems from a more realistic inclusion of these events in the model framework.

Published as:

Klein, Tony (2017): Dynamic Correlation of Precious Metals and Flight-to-Quality in Developed Markets, in: Finance Research Letters, Vol. 23C, pp. 283-290, DOI: 10.1016/j.frl.2017.05.002.

Presented and discussed at:

- HypoVereinsBank PhD Seminar Spring 2017, Berlin

2.7 Trend Contagion in WTI and Brent Crude Oil Spot and Futures Prices - A Spread and Correlation Analysis

Referenced as: Paper 7 (Trends)

Abstract

This article examines the interconnectedness of WTI and Brent prices on different resolutions of price movements. Firstly, within a multivariate BEKK framework we identify high but volatile correlations with recurring highs around 0.8 and multiple periods of decoupling. Meetings of the Organization of the Petroleum Exporting Countries (OPEC) increase the correlation. Secondly, linear ℓ_1 trends reveal that long-term movements of WTI and Brent are driven by the same dynamics, confirming the ‘globalized market’ hypothesis. OPEC meetings have only little impact on long-term price trends. Thirdly, we find leading effects of WTI over Brent by short-term trends of several days, especially in negative direction. These trends have an asymmetrical effect on volatility; negative trends cause a stronger increase than positive trends. These findings are of interest to policy makers as well as hedging strategies of crude oil portfolios and provide insight to long-term movements of crude prices. We provide evidence that OPEC meetings cause short-term market reactions but have little effect on long-term price developments.

Published as:

In its present version: forthcoming in the proceedings of the 40th IAEE International Conference 2017, Singapore; usage as *job market paper* (work in progress)

Presented and discussed at:

- 5th International Symposium on Environment and Energy Finance Issues (ISEFI) 2017, Paris, France
- 40th IAEE International Conference 2017, Singapore

Awards

- Student Paper Award, 40th IAEE International Conference 2017, Singapore

2.8 Other Articles

Other work that has been published during the course of the author’s doctoral studies which is not part of this dissertation:

LOCAREK-JUNGE, HERMANN; KLEIN, TONY; WALTHER, THOMAS (2014):
GARCH-Modelle, in: WISU - Das Wirtschaftsstudium, Vol. 43, Nr. 11, S. 1348-1354, 1387.

WALTHER, THOMAS; KLEIN, TONY (2015): Contingent Convertible Bonds and its Impact on Risk-Taking of Managers, in: Cuadernos de Economía - Spanish Journal of Economics and Finance, Vol. 38, Issue 106, pp. 54-64, DOI: 10.1016/j.cesjef.2014.09.001.

LOCAREK-JUNGE, HERMANN; KLEIN, TONY (2016): Währungs-Hedging mit Derivaten, in: WISU - Das Wirtschaftsstudium, Vol. 45, Nr. 3, S. 298 - 304, 335.

2.9 Topical Relatedness

Calculation times of the parameter estimation for the Mixture Memory GARCH in Paper 1 (MM) were extremely slow and two issues were found to increase calculation times. Firstly, the iterative structure of the EM-algorithm consumed significant amounts of time. This is accelerated by adding parameter estimates of the previous iteration as starting values to the optimization procedures as well as a parallel computation of the separated maximizations. Secondly, the look-back or long memory property of FIGARCH and FIAPARCH is extremely costly regarding the number of calculations. This is dependent on the implemented truncation lag, which is further examined in Subsection 3.1.1.

We address the latter issue raised above by introducing fast fractional differencing to variance modeling in Paper 2 (FF). We demonstrate how calculation times of FIGARCH and FIAPARCH are accelerated by factor up to 40 and up to 400, respectively. For example, the rolling window, out-of-sample analysis carried out in Paper 1 (MM) for FIGARCH is reduced from 67 minutes to roughly 3 minutes calculation time. Ultimately, calculation times for all long memory models implemented in the papers presented herein, benefit from this new calculation scheme.

Paper 1 (MM) also motivated Paper 3 (FX). For both crude oil blends, we identify several econometric properties of the return series such as varying long memory and asymmetric response to shocks in variance. We examine the EUR/PLN exchange rate and test if and to what extend these properties are present. Fast fractional differencing is used for long memory and we find significant effects.

The lack of literature on exchange rates of Eastern-European, non-EMU countries and the results for the EUR/PLN exchange rate in Paper 3 (FX) motivated Paper 4 (LM). We focus on the long memory property and examine if it is a true or spurious phenomenon across different currency pairs to the Euro. With a variety of tests and modifications of variance models accounting for variance shifts, Paper 4 (LM) offers an overview of up-to-date techniques and advances in this particular field of research.

Paper 5 (PM) addresses an issue present in all previous four papers; the choice of

proxy for the realized or true daily variance which is applied in loss functions as measure of goodness-of-fit and forecasting performance. Another class of commodities—precious metals—is analyzed and intra-day data is used to proxy realized volatility in a more realistic way. This article features univariate models as well as a multivariate framework to model correlations.

Given the findings of Paper 5 (PM) regarding the econometric properties such as emphasized asymmetry in variance for Gold and Silver, the DCC model is extended to account for the properties of asymmetry and long memory in the asset-specific variances in Paper 6 (PM-Corr). The flight-to-quality phenomenon is addressed with the adjusted DCC for different indexes and precious metal pairs.

The differences of WTI and Brent crude oil, both in price levels and in econometric properties such as long memory behavior presented in Paper 1 (MM), are further examined in Paper 7 (Trends). Contagion effects in short and long term trends of spot and futures prices are detected with a variety of approaches. One of these approaches is the BEKK framework for correlation modeling as an methodological alternative to the DCC applied and extended in Paper 6 (PM-Corr).

Figure 1 visualizes the field of application (commodities or exchange rates), the model design (univariate or multivariate), and the linkages between the essays.

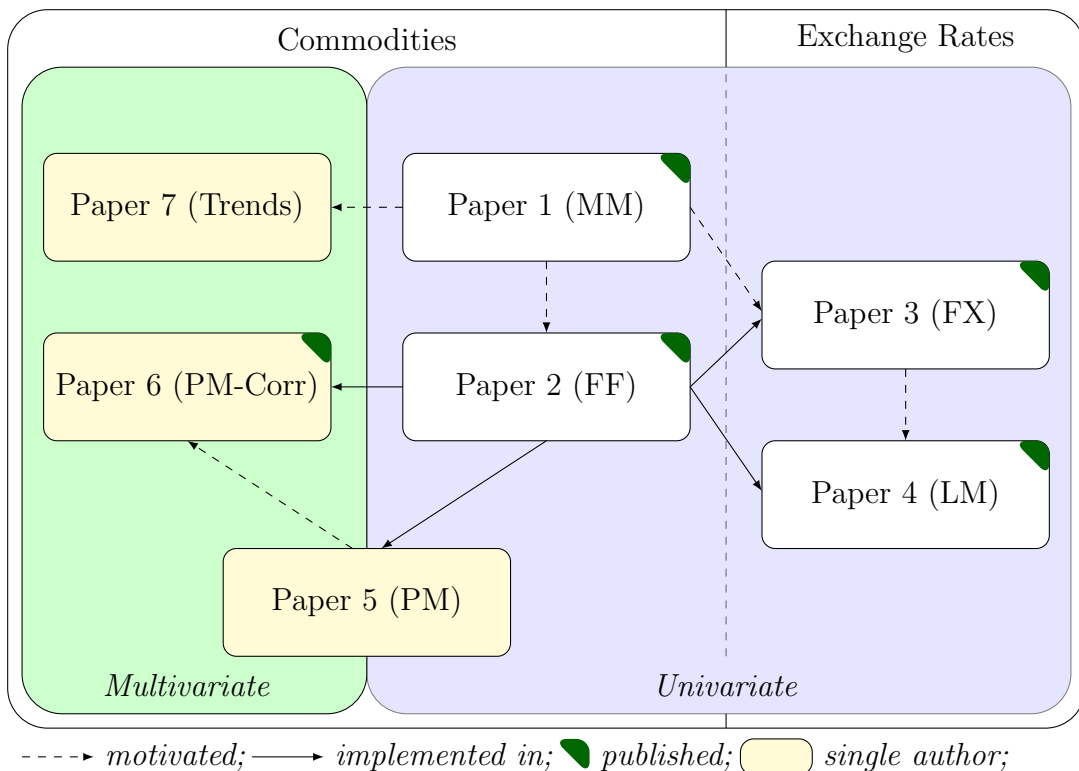


Figure 1: Interconnectedness of all published articles and working papers which are part of the present cumulative dissertation.

3 Selected Topics

3.1 Application of Fast Fractional Differencing

This section is based on the methodology presented in Klein & Walther (2017) and presents additional results which have not been published with this paper. To some extent, the following sections were part of an earlier, unpublished manuscript⁵ and composed solely by the author of this thesis.

3.1.1 Methodological Motivation

One of the first to analyze long memory fractional noise are Lawrance & Kottegoda (1977) who examine river flows from a hydrologically motivated perspective. Interestingly, the same issue of river flow modeling was addressed earlier in an article of Hurst (1951) which yields a widely used test for long range dependent dynamics. Hosking (1981) identifies the lack of a proper model formulation for the long memory property and extends the Autoregressive Integrated Moving Average model of Box & Jenkins (1976, ch. 4, 5) by allowing for a real valued degree of differencing. Henceforth, this characterization is referred to as fractional differencing.

The fractional difference $(1 - L)^d$ of a time series $\{x_t\}_{t=1}^n$ is expressed as a binomial expansion (Hosking, 1981)

$$(1 - L)^d x_t = \sum_{i=1}^{\infty} \pi_i(d) x_{t-i}, \quad (1)$$

where L is the lag operator with the real-valued, fractional difference parameter d and

$$\pi_i(d) = \frac{-d(1-d) \cdots (i-1-d)}{i!}, \quad (2)$$

for all i with $\pi_i(d) \xrightarrow{i \rightarrow \infty} 0$.

Fractional differencing, as an effective tool to model slowly decaying autocorrelation, is found in a plethora of econometric models. The most important example is the *Autoregressive Fractionally Integrated Moving Average* (ARFIMA) which is introduced by Granger & Joyeux (1980) and extended in Hosking (1981). ARFIMA models are applied for a wide range of problems that goes far beyond financial time series. Climate modeling (Baillie & Chung, 2002) or cell phone data and molecular biology (Burnecki, 2012, ch. 6) are a few examples. In financial time series analysis with focus on variance modeling, FIGARCH (Baillie et al., 1996) or FIAPARCH (Tse,

⁵The unpublished version was titled “On the Application of Fast Fractional Differencing in Modeling Long Memory of Conditional Variance.”

1998) are widely-used frameworks that incorporate fractional differencing.⁶

Applying fractional differencing in our framework of long memory conditional variance modeling, the infinite summation in Eq. (1) has to be finite and as such, truncated at a certain point. This requires a truncation lag $n_{\text{trunc}} \in \mathbb{N}$ that terminates the convolution sum of the differencing term and the time series. As the truncation lag terminates the summation, it is of essential practical importance in two ways.

The first matter to consider is an increasing computation time when increasing the truncation lag. Naturally, more observations and weights depending on model parameters in the convolution require more total computations. We address the theoretical Landau symbols ($\mathcal{O}(\cdot)$, see further: Landau, 1909) of the calculations in Klein & Walther (2017, p. 276). This is of interest for parameter estimations, since they require a vast amount of repetitive calculations with different parameter choices. We give some examples in the simulation section.

The second issue with choosing the truncation lag arises from the actual definition of the fractional difference term and its application in long memory models. Simpler GARCH models feature a fast decline of the impact of shocks on the conditional variance due to their exponential decay. In order to overcome this short memory, the fractional differencing term is introduced and defines the Fractionally Integrated GARCH, which features a hyperbolic decay of shocks. Fixing a truncation lag has direct impact on the memory structure of the given model. If the truncation lag is chosen too low, the long memory property is not correctly depicted yielding insufficient estimates (Baillie et al., 1996, p. 12f.; Stoev & Taqqu, 2004, p. 116).

For reasons of simplicity, we define the following framework. Let $\{\varepsilon_t\}_{t=1}^n$ denote a return (or residual) series with zero mean and total number of observations $n \in \mathbb{N}$. The conditional variance of ε_t , which is denoted σ_t^2 , is then given by

$$\begin{aligned}\varepsilon_t &= \sigma_t z_t, \\ \mathbb{E}(\varepsilon_t^2 | \mathcal{F}_{t-1}) &= \sigma_t^2,\end{aligned}\tag{3}$$

for each $t = 1, \dots, n$, where $z_t \sim \mathcal{N}(0, 1)$ i.i.d.⁷ with the sigma algebra \mathcal{F}_{t-1} generated by the past of the series (returns as well as conditional variances) up to time $t-1$. For the conditional variance σ_t^2 , we implement two long memory models to demonstrate the advantages of fast fractional differencing; firstly, the FIGARCH(1, d ,1) defined in

⁶A list of additional examples is given in Klein & Walther (2017, p. 278).

⁷Alternative distributions and frameworks with time-varying mean, such as AR or ARMA structures, are realized in the papers summarized in this cover paper.

Baillie et al. (1996):

$$\begin{aligned}
\sigma_t^2 &= \omega + \left(1 - \beta L - (1 - \phi L)(1 - L)^d\right) \varepsilon_t^2 + \beta \sigma_{t-1}^2 \\
&= \frac{\omega}{1 - \beta} + \left(1 - \frac{(1 - \phi L)(1 - L)^d}{1 - \beta L}\right) \varepsilon_t^2 \\
&= \frac{\omega}{1 - \beta} + \sum_{i=1}^{\infty} \lambda_i \varepsilon_{t-i}^2,
\end{aligned} \tag{4}$$

where

$$\begin{aligned}
\lambda_1 &= \phi - \beta - d, \\
\lambda_i &= \beta \lambda_{i-1} + \left(\frac{i-1-d}{i} - \phi\right) \left(\frac{(i-2-d)!}{i!(1-d)!}\right),
\end{aligned} \tag{5}$$

where λ_i are FIGARCH weights given in Bollerslev & Mikkelsen (1996). The sufficient, non-negativity constraints $\omega > 0$, $0 \leq \beta \leq \phi + d$, and $0 \leq d \leq 1 - 2\phi$ limit admissible parameter choices (see further: Klein & Walther, 2017, p. 275, and references therein).

Secondly, we implement the FIAPARCH of Tse (1998), which is a FIGARCH applied on APARCH (Ding et al., 1993) innovations. Hence, the model is defined analogously by

$$\begin{aligned}
\sigma_t^\delta &= \omega + \left(1 - \beta L - (1 - \phi L)(1 - L)^d\right) (|\varepsilon_t| - \gamma \varepsilon_t)^\delta + \beta \sigma_{t-1}^\delta \\
&= \frac{\omega}{1 - \beta} + \sum_{i=1}^{\infty} \lambda_i (|\varepsilon_{t-i}| - \gamma \varepsilon_{t-i})^\delta,
\end{aligned} \tag{6}$$

where λ_i are calculated as in Eq. (5) and γ is the leverage parameter with $\gamma \in (-1, 1)$. The power parameter $\delta > 1$ is a Box-Cox transformation of the volatility. Parameter restrictions for FIAPARCH are dependent on the underlying distribution of z_t . For a discussion on this matter, we refer to Ding et al. (1993, pp. 103-105), Tse (1998, p. 54), and Klein & Walther (2016, p. 49). Given the more complex computational nature of the FIAPARCH, we include both long memory models in the comparison. The last lines of Eq. (4) and Eq. (6) are referred to as ARCH(∞)-representation of the respective model.

Calculating the fractional difference of the time series with weights given in Eq. (5), a truncated, linear convolution of the residual series ($\{\varepsilon_t^2\}$ for FIGARCH or $\{(|\varepsilon_t| - \gamma \varepsilon_t)^\delta\}$ for FIAPARCH) and the corresponding weights λ_i is performed, which reads

$$\sigma_t^2 = \frac{\omega}{1 - \beta} + \sum_{i=1}^{n_{\text{trunc}}} \lambda_i \varepsilon_{t-i}^2, \tag{7}$$

for FIGARCH according to Eq. (4).

A truncation lag of $n_{\text{trunc}} = 1\,000$ indicates that the preceding 1 000 observations (with indexes $t - 1, \dots, t - 1000$) are used in the convolution sum at time t of the fractional difference to depict the long memory property. Following this example, the first $\tilde{t} = 1, \dots, 999$ observations would require $(\varepsilon_{-999}^2, \dots, \varepsilon_{-1}^2, \varepsilon_0^2)$ or subsets to be known⁸ in order to apply the convolution as in Eq. (7).

There are different approaches to this matter. Firstly, sometimes applied in ARFIMA models, only ε_t^2 with $t > 0$ are included in the convolution sum. Hence, the truncation lag for the first $\tilde{t} = 1, \dots, 999$ calculations equals that index and is increasing with each observation up to the chosen truncation lag (Jensen & Nielsen (2014) apply this technique). Secondly, the first n_{trunc} observations are only included for calculation of the long memory convolution sum from $t = n_{\text{trunc}} + 1$. These so-called pre-sample observations are extensively discussed in Johansen & Nielsen (2012). Lastly, a constant back-cast of length n_{trunc} is calculated from the observations $\{\varepsilon_t^2\}_{t=1}^n$ and augmented to the residual vector, extending the latter to length $n + n_{\text{trunc}}$. This so-called augmentation is commonly used in modeling long memory of conditional variance. Choices for the back-cast range from the unconditional variance of the observations to more complicated calculations which are dependent on the model implemented. A discussion on this matter can be found in Francq & Zakoian (2004, pp. 208f.).

The performance of three different ways of calculating the convolution sum in Eq. (4) and Eq. (6) are compared. In order to obtain results with practical relevance, we time a calculation of a full series $\{\sigma_t^2\}_{t=1}^n$ by the following approaches:⁹

| | |
|-------------------------------------|---|
| <i>Iterative Summation</i> (ItS) | Naive element-wise multiplication and summation in an iteration over the chosen truncation lag, |
| <i>Vectorized Summation</i> (VS) | MATLAB -optimal ¹⁰ vectorization of the summation, and |
| <i>Fast Fourier Transform</i> (FFT) | Calculation of the convolution sum with the FFT approach. |

ItS is only included to serve as a worst-case scenario for the run-time measurements and it is not recommended to implement it in any application of the above-mentioned conditional variance models. We make use of the pre-implemented **MATLAB** functions `fft` and `ifft` which translate to the fast Fourier transform algorithm and its inverse.

⁸This refers to observations prior to the sample period with $t \in \mathbb{Z}$.

⁹A comparison of the estimation times for each model is carried out in Klein & Walther (2017) with sample lengths up to $n = 40\,000$.

¹⁰**MATLAB** is optimized to perform operations on matrices and vectors faster than scalar-based operations. See further: http://www.mathworks.com/help/matlab/matlab_prog/vectorization.html.

3.1.2 Convolution Sum Timing

In order to achieve process-specific results, the first step is to generate a FIGARCH process to provide realistic $\{\varepsilon_t\}_{t=1}^n$. Given these residuals, we calculate the augmented residuals needed for applying long memory on the first data points. As the last step before timing, we calculate the weights λ_i for $i = 1, \dots, n_{\text{trunc}}$ according to Eq. (5) for a fixed set of parameters. All necessary inputs for the implementations of the different approaches presented in the previous section are known. We then time the execution of the scripts to calculate the full conditional variance series $\{\sigma_t^2\}_{t=1}^n$.

In order to obtain robust results, the analysis is repeated for different sample lengths $n \in \{5\,000, 7\,500, 10\,000, 15\,000\}$ as well as for 16 different truncation lags ranging from 50 to 4 000. This way, 64 different combinations are tested and each implementation is timed for $N = 1\,000$ repetitions.¹¹

Figure 2 plots the mean calculation times for different lags. Each subplot presents a specific sample length given in its title. From the plots, it is easy to see that the FFT approach clearly outperforms the ItS and VS implementation. For example, at a truncation lag of $n_{\text{trunc}} = 1\,000$ and a sample length of $n = 15\,000$, ItS needs 0.1788s, VS takes 0.1356s while FFT only requires 0.0033s to fully calculate $\{\sigma_t^2\}_{t=1}^n$. While these times are generally low, note that it includes only one calculation of the conditional variance series. This time difference is of greatest interest in parameter estimations where thousands of calculations of the variance are realized.

For FFT at higher lags (500 and onward), the sample length appears to have almost no practical impact on calculation times. For example, the FFT calculation time for $n_{\text{trunc}} = 1\,000$ are 0.0020s, 0.0034s, 0.0035s, and 0.0036s for the respective sample lengths of 5 000, 7 500, 10 000, and 15 000. Table 1 provides an overview of calculation times for a sample length of $n = 15\,000$.

| | Lags | | | | | | |
|-----|--------|--------|--------|--------|--------|--------|--------|
| | 500 | 750 | 1 000 | 1 500 | 2 000 | 3 000 | 4 000 |
| ItS | 0.1787 | 0.2450 | 0.3117 | 0.4459 | 0.5778 | 0.8484 | 1.1220 |
| VS | 0.1356 | 0.1453 | 0.1575 | 0.1814 | 0.2023 | 0.2727 | 0.3201 |
| FFT | 0.0032 | 0.0034 | 0.0035 | 0.0089 | 0.0120 | 0.0125 | 0.0145 |

Table 1: Mean computation time in seconds for a series of length $n = 15\,000$ (FIGARCH) with $N = 1\,000$ repetitions.

Comparing the speed-up of the FFT relative to the time needed by ItS and VS, the difference of the approaches becomes very clear. Tab. 2 presents these results for $n = 15\,000$. Much to our surprise, the speed-ups drop significantly for truncation lags of 1 500 and 2 000 while for 3 000 and 4 000 they increase again. A dropping speed-up is

¹¹In the tables, we only report results for $n_{\text{trunc}} \geq 500$ as lower truncation lags are barely implemented in the application of these models. We include these short lags for the sole purpose of comparison.

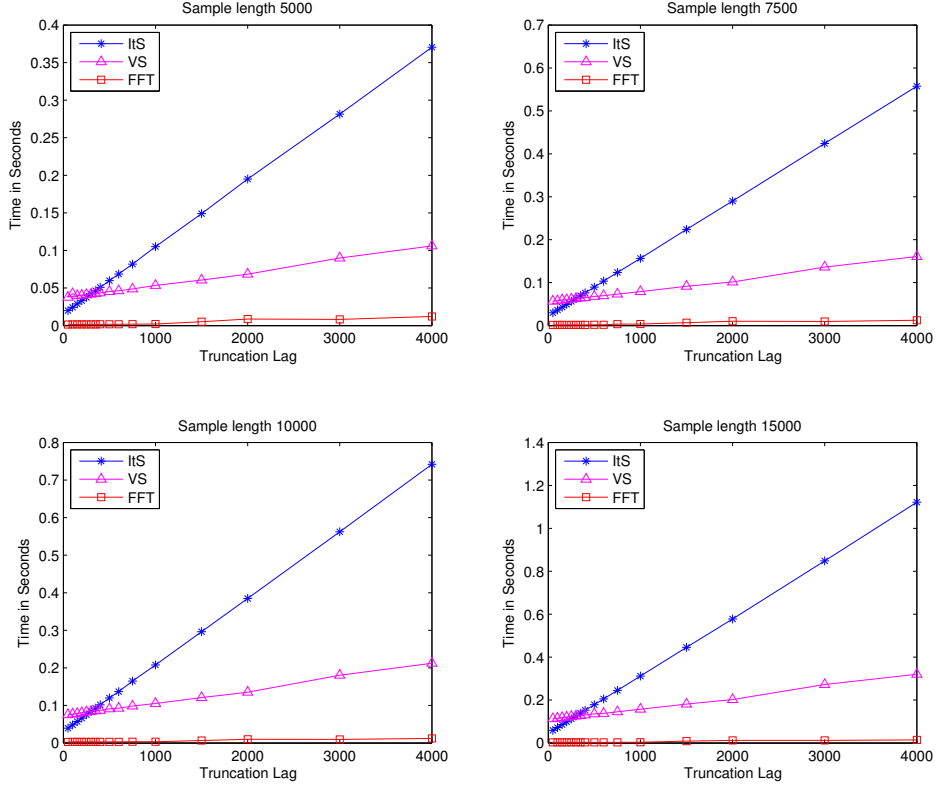


Figure 2: Mean computation time in seconds to calculate $\{\sigma_t^2\}_{t=1}^n$ of different sample lengths with a given set of parameters and $\{\varepsilon_t\}_{t=1}^n$. The truncation lag (n_{trunc}) describes the length of the fractional differencing term.

caused by an increased FFT computation time relative to the corresponding ItS or VS time. We repeated the above mentioned calculations for different sets of parameters generating the ε_t which is fed into the calculations. In addition, the experiments were carried out on two additional machines. Naturally, the computation times differed slightly; the drop in speed-ups, however, is persistent over all experiments. A possible explanation might be the numerical handling of the `fft` and `ifft` function in `MATLAB` as well as memory allocations for different array sizes. The truncation lag size of 1 500 is right in between two powers of 2 ($2^{10} = 1\,024$ and $2^{11} = 2\,048$), which would explain some of the decrease in speed-up for this lag as it has immediate impact on the fast Fourier transform. This does not explain the drop for $n_{\text{trunc}} = 2\,000$, however.

| | Lags | | | | | | |
|-----|------|------|-------|-------|-------|-------|-------|
| | 500 | 750 | 1 000 | 1 500 | 2 000 | 3 000 | 4 000 |
| ItS | 55.0 | 71.9 | 87.7 | 49.9 | 46.4 | 70.6 | 77.1 |
| VS | 41.7 | 42.6 | 44.3 | 20.3 | 16.3 | 22.7 | 22.0 |

Table 2: Mean speed-ups of FFT relative to the respective computation times for a sample length $n = 15\,000$.

3.1.3 Numerical Precision of Different Calculation Schemes

The numerical precision of the fast Fourier transform algorithm and its computer implementation are very well predictable. The root-mean-square relative error (or L_2 relative error) of the Cooley & Tukey (1965) algorithm as well as newer extensions feature an $\mathcal{O}(\sqrt{\log_2 n})$ growth rate. For example, Frigo & Johnson (2005) (on whose work the `fft` function in `Matlab` is based on) and references therein calculate errors of around 1.5×10^{-16} for the application of the fast Fourier transform of samples of size $n = 2^{14} = 16\,384$. For more discussion on errors of different FFT implementations, it is referred to Schatzman (1996).

For the implementation of fast fractional differencing in modeling the conditional variance, we control for the following difference measures: the maximum or Chebyshev vector norm and L_2 vector norm applied on the difference of the variance calculated by the VS and FFT approach:

$$\begin{aligned} \|\sigma_{\text{VS}}^2 - \sigma_{\text{FFT}}^2\|_{\infty} &= \max_{t=1,\dots,n} |\sigma_{\text{VS},t}^2 - \sigma_{\text{FFT},t}^2|, \\ \|\sigma_{\text{VS}}^2 - \sigma_{\text{FFT}}^2\|_2 &= \left(\sum_{i=1}^n (\sigma_{\text{VS},i}^2 - \sigma_{\text{FFT},i}^2)^2 \right)^{\frac{1}{2}}, \end{aligned} \tag{8}$$

where σ_{VS}^2 and σ_{FFT}^2 refer to the full series of the variance obtained with the respective calculation scheme. The numerical values offer evidence of the precision of the new algorithm for this model class and practical relevance of possible differences of the implementations.

We analyze $N = 1\,000$ repetitions for all lag/length combinations from this experiment and calculate the different norms given in Eq. 8 separately. The mean of the differences is calculated and presented in Figure 3. Firstly, we find the differences under the given norms to be in line with findings of Frigo & Johnson (2005) with respect to the dimension of the errors. The mean maximum difference measured by the L_{∞} norm takes values between 0.2×10^{-15} and 2.0×10^{-15} , while the error increases with the expected rate. Secondly, for the L_2 or Euclidean norm the differences are of the same dimension while—expectedly—increasing with an increasing truncation lag. Neither for the L_{∞} nor the L_2 norm, there is an obvious relationship to the sample size, underlining the fact that these numerical errors are directly connected with and close to the machine epsilon for double precision floating point numbers.

It follows that there is virtually no numerical difference in sense of practical application between the VS and FFT calculation of $\{\sigma_t^2\}_{t=1}^n$. Since the difference between σ_{VS}^2 and σ_{FFT}^2 is negligibly small compared to their values, we find parameter estimates to be indistinguishable between the two approaches.

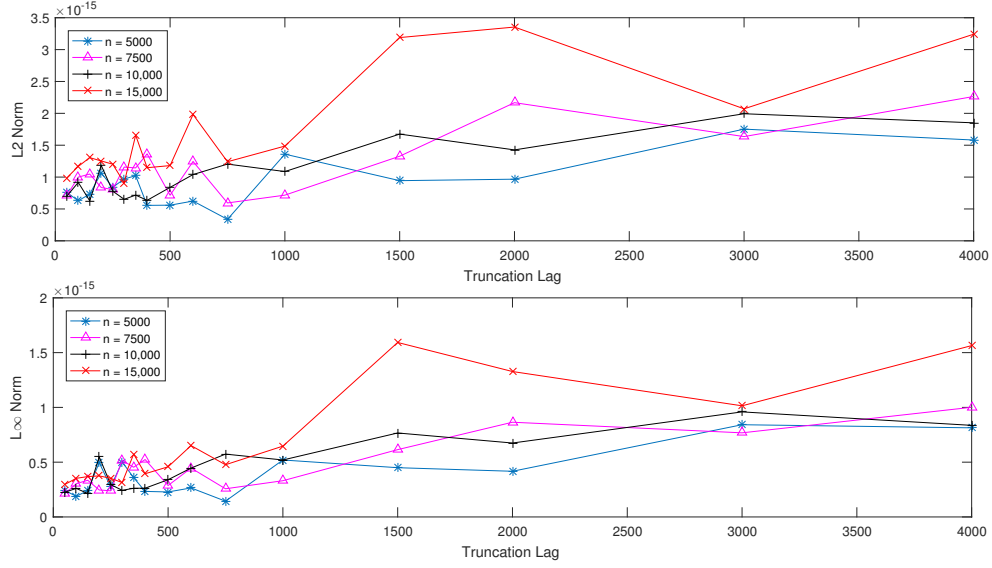


Figure 3: Mean difference of σ_{VS}^2 and σ_{FFT}^2 under the L_2 and L_∞ norm for different sample lengths and truncation lags. Each length/lag combination carried out with $N = 1000$ repetitions.

3.1.4 Application in Rolling-Window Variance Forecasting

In order to further demonstrate the computational advantage of the FFT approach, the VS and FFT implementations are applied on crude oil returns that feature a relatively long memory in their conditional variance, which emphasizes the relation of Paper 1 (MM) and Paper 2 (FF). Since the U.S. WTI and European Brent returns differ in their structure featuring a varying memory structure (Klein & Walther, 2016), we compare the performance of the implementations separately. The spot prices are obtained from the U.S. Energy Information Administration (EIA) and transformed to log-returns.¹² For both blends, prices from 03-Jan-1986 to 31-Dec-2015 are utilized.

We demonstrate the effectiveness of the FFT approach by a rolling-window estimation over five years, 03-Jan-2011 to 31-Dec-2015, as out-of-sample period. We fix the training sample length at $n_{\text{sample}} = 5500$. This equals $K_W = 1261$ separate computations for WTI and $K_B = 1259$ separate estimations for Brent yielding five years of daily conditional variances. Rolling-window estimations are often needed for out-of-sample model testing or—equally important—in risk management where the conditional variance is utilized in Value-at-Risk forecasts over a specific time frame (regarding this dissertation, see further: Klein & Walther, 2016, pp. 50-57).

Accounting for the complexity of a real-world application, we time the whole process of a parameter estimation which includes: finding starting parameters,¹³ pa-

¹²Let P_t and P_{t-1} denote consecutive prices. The resulting log return r_t is calculated by $r_t = \log P_t - \log P_{t-1}$ and $r_t = \mu_t + \varepsilon_t$ as defined in Eq. 3.

¹³Finding starting values is carried out with a grid search approach by comparing log-likelihoods. This also includes the VS or FFT approach.

parameter transformations, log-likelihood maximization, and calculation of the final conditional variance as well as log-likelihood.

Figure 4 plots the total computation time for the out-of-sample analysis of FI-GARCH for WTI.¹⁴

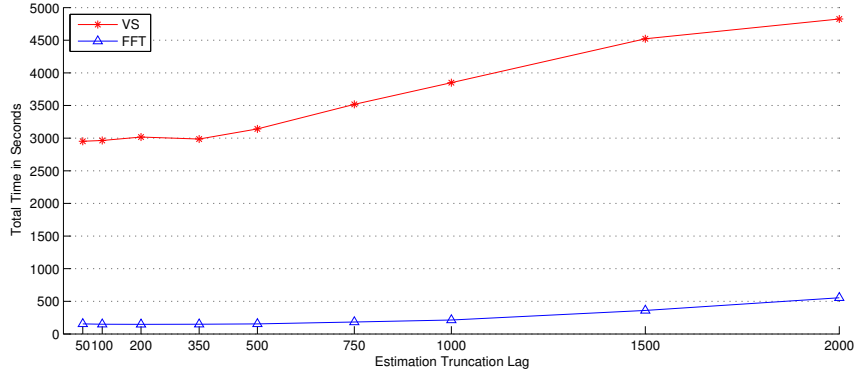


Figure 4: Total times for $K_W = 1261$ sequential out-of-sample FIGARCH parameter estimations for different lags for WTI (FIGARCH). Vectorized summation is plotted in red; the Fast Fourier Transform approach is plotted in blue.

For a large-scale truncation lag of $n_{\text{trunc}} = 2000$, the computation time of an out-of-sample analysis of Brent over $K_B = 1259$ estimations is reduced from 4782s (approx. 79.7 minutes) with vectorized summation to 413s (approx. 6.9 minutes) with application of FFT. Results for the WTI differ only slightly as the following calculation of the speed-ups shows. We achieve speed-ups of factor 11.6 to 21.1 for Brent and of factor 8.7 to 20.5 for WTI with FFT over VS for the rolling-window analysis described above. The speed-ups for all lags are given in Table 3. Notably, the same phenomenon of dropping speed-ups as reported in Subsection 3.1.2 for lags of order 1500 and 2000 is observable.

| | Lags | | | | | | | | |
|-------|------|------|------|------|------|------|------|------|------|
| | 50 | 100 | 200 | 350 | 500 | 700 | 1000 | 1500 | 2000 |
| Brent | 15.6 | 20.1 | 21.1 | 19.5 | 20.5 | 19.3 | 20.4 | 16.7 | 11.6 |
| WTI | 19.1 | 19.8 | 20.5 | 20.0 | 20.3 | 19.2 | 17.9 | 12.6 | 8.7 |

Table 3: Factor of speed-ups of FFT compared to VS for an out-of-sample analysis of FIGARCH.

The acceleration for the FIAPARCH estimation is even more pronounced. As we find it unfeasible to carry out an out-of-sample analysis over K_B and K_W points of the FIAPARCH with the VS approach, we present the significant difference of a single estimation for different lags. Consistently to the analysis above, we choose the last out-of-sample point with $n_{\text{sample}} = 5500$ and record the time for a full estimation of

¹⁴Since a rolling window analysis is suitable for parallelization, we implement a `parfor`-loop in `MATLAB` over K for both approaches to carry out the study in a feasible amount of time.

parameters and calculation of the final conditional variance. The results are given in Figure 5.

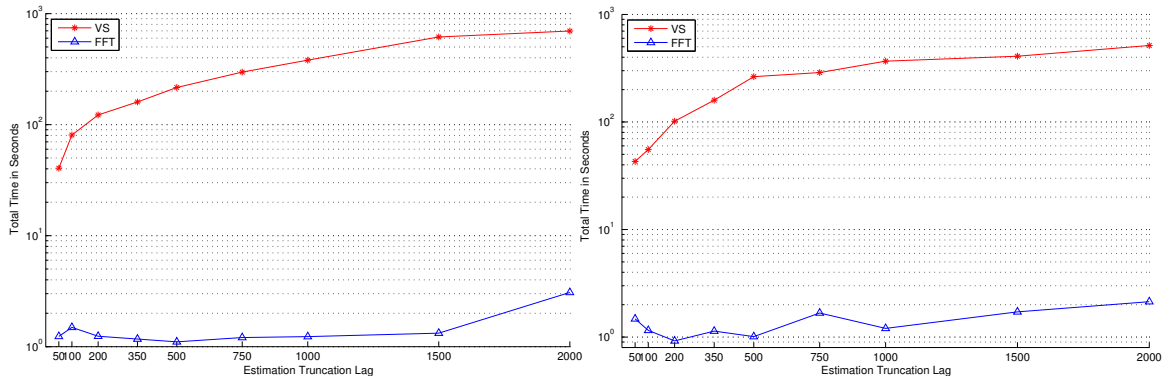


Figure 5: Parameter estimation times in log scale for different truncation lags for WTI (left) and Brent (right) for FIAPARCH for a single estimation. Data with $n_{\text{sample}} = 5500$ is taken from the last out-of-sample point of the rolling window estimation.

Both the VS and FFT approach yield identical parameters and log-likelihood. The calculation time of our VS FIAPARCH implementation is extremely slow and seems rather inefficient compared to the VS FIGARCH. With a chosen truncation lag of $n_{\text{trunc}} = 1000$, the complete parameter estimation using vectorized summation consumes 380s (approx. 6.3 minutes) while the FFT approach only takes 1.2s. Comparing the calculation times for $n_{\text{trunc}} = 2000$, we accelerate the estimation from 696s (around 11.6 minutes) to 3.1s. For this single estimation, we achieve very high speed-ups which are given in Table 4.

| | Lags | | | | | | | | |
|-------|------|------|-------|-------|-------|-------|-------|-------|-------|
| | 50 | 100 | 200 | 350 | 500 | 700 | 1000 | 1500 | 2000 |
| Brent | 32.8 | 54.0 | 98.5 | 136.4 | 195.3 | 246.0 | 308.9 | 388.0 | 155.1 |
| WTI | 29.1 | 48.0 | 110.5 | 140.0 | 261.1 | 171.9 | 305.8 | 238.1 | 240.8 |

Table 4: Factor of speed-up of FFT compared to VS for a single parameter estimation of FIAPARCH.

The results for modeling the conditional variance of WTI and Brent returns with FIGARCH and FIAPARCH show clearly that the FFT approach offers a valuable speed-up compared to traditional summations of the fractional differencing term.

For this rolling-window estimation based on real data, we also find that increasing the lag to $n_{\text{trunc}} = 3000$ or 4000 does not necessarily imply that the goodness-of-fit, measured with the log-likelihood, increases as well, which is in sharp contrast to the simulation study with true long memory DGPs. This could have different reasons. Some important studies on crude oil spot and futures prices have revealed different volatility regimes. Nomikos & Pouliasis (2011) find that Mix-GARCH and Regime-Switching-GARCH models have better goodness-of-fit as well as forecasting quality

than single-regime models. This supports earlier results by Fong & See (2002) who find regime switching models to generally perform better than non-switching approaches. Chang (2012) incorporates asymmetric effects in regime switching models that provide a better fit. Besides the relative slow decay of shocks in oil prices, these studies show that the conditional variance features regimes of different levels. This could cause a large truncation lag on the residuals to bias the conditional variance as the weights λ_i are unable to distinguish between regimes. Given the long look-back period, these weights might span over different regimes. This issue is closely related to Paper 4 (LM), where switches are accounted for. Walther et al. (2017) only focus on exchange rates, however. Additionally, a large lag implies an augmentation of the same length. As we face a relatively short rolling-window of length $n = 5\,500$ observations, large lags might bias the parameter estimation by adding a constant term for a long period prior to the first observation of the time series. Pre-sample observations would counter this effect but lead to reduction of total observations available, if daily returns are examined.

3.2 Properties of Commodity Spot and Futures Returns

This section briefly extends and connects the findings of Paper 5 (PM), Paper 6 (PM-Corr), and Paper 7 (Trends) and presents supporting results.

3.2.1 Precious Metals—On Assets and Industrial Metals

Precious metals such as Gold, Silver, Platinum, and Palladium are often focused on in academic literature as they serve multiple purposes. In general, these metals could be categorized as an asset or as an industrial metal depending on the demand. Gold is clearly treated as an asset rather than an industrial metal (Hammoudeh et al., 2010, p. 640), as the industrial demand is low compared to the demand generated by private and institutional investors, countries, central banks, and applications in jewelery and art. Silver features similar properties, the industrial demand is much higher, however. Also, if Silver is used as asset, storage cost plays a major role because the price of Silver is only a fraction of that of Gold and other precious metals. Palladium and Platinum are classical industrial metals (an extensive literature analysis is found in Vigne et al., 2017). The automobile industry generates a large portion of the demand (Alonso et al., 2012, pp. 12986f.; Massari & Ruberti, 2013, p. 36). In the literature, it is agreed on that these four precious metals are very sensitive to changes in demand and supply (Batten et al., 2010, p. 70; Arouri et al., 2012, p. 208). In addition, the empirical distribution of the returns of these precious metals differs which is outlined in Batten & Lucey (2010, pp. 68f.), Hammoudeh et al. (2011, pp. 237f.), and Klein (2017, pp. 286f.) among others. The application of these metals implies

distinct features of their prices and variances. Gold—as an safe haven asset¹⁵—reacts differently to market shocks or ongoing turmoil than precious metals with an industrial purpose. This is described in detail in the following paragraphs.

Aiming to identify these differences, we apply a variety of variance models with an AR(1) mean specification on spot and futures prices from 01-Jan-2000 to 31-Dec-2016. The data is taken from the data set processed in Klein (2017). In terms of the conditional variance framework defined in Eq. (3), it is augmented to

$$\begin{aligned}\varepsilon_t &= \mu_0 + \mu_1 \varepsilon_{t-1} + \sigma_t z_t, \\ \mathbb{E}(\varepsilon_t^2 | \mathcal{F}_{t-1}) &= \sigma_t^2,\end{aligned}\tag{9}$$

with $z_t \sim \text{St-}t_\nu(0, 1)$ i.i.d. and $\{\sigma_t^2\}_{t=1}^n$ is modeled as GARCH, APARCH, FIGARCH (abbr. FIG), and FIAPARCH (abbr. FIAP). Table 5 and Table 6 list parameter estimates for spot and three months (3M) futures price returns of Gold and Silver, respectively. Table 7 lists the estimates for Palladium and Platinum spot price returns.¹⁶ The following tables provide the choice of models included in the adjusted DCC in Klein (2017). From these tables, the following can be summarized for the sample period of 2000 to 2016.

| | Gold spot ($n = 4431$) | | | | Gold GC3 ($n = 4268$) | | | |
|----------|--------------------------|---------------------|---------------------|---------------------|-------------------------|---------------------|---------------------|---------------------|
| | GARCH | APARCH | FIG | FIAP | GARCH | APARCH | FIG | FIAP |
| μ_0 | 0.0358 (0.0132) | 0.0423 (0.0132) | 0.0348 (0.0147) | 0.0414 (0.0130) | 0.0443 (0.0137) | 0.0488 (0.0137) | 0.0429 (0.0140) | 0.0473 (0.0141) |
| μ_1 | -0.0515 (0.0144) | -0.0511 (0.0139) | -0.0478 (0.0108) | -0.0535 (0.0136) | -0.0368 (0.0136) | -0.0394 (0.0128) | -0.0353 (0.0138) | -0.0401 (0.0141) |
| ω | 0.0130 (0.0043) | 0.0109 (0.0034) | 0.1301 (0.0011) | 0.2386 (0.0396) | 0.0135 (0.0045) | 0.0116 (0.0041) | 0.1452 (0.0367) | 0.2818 (0.0356) |
| α | 0.0485 (0.0083) | 0.0472 (0.0054) | | | 0.0426 (0.0072) | 0.0443 (0.0050) | | |
| β | 0.9435 (0.0098) | 0.9528 (0.0074) | 0.7086 (0.0083) | 0.9255 (0.0138) | 0.9488 (0.0091) | 0.9562 (0.0072) | 0.6931 (0.0647) | 0.9290 (0.0347) |
| ϕ | | | 0.2277 (0.0023) | 0.0000 (0.0000) | | | 0.2484 (0.0416) | 0.0003 (0.0004) |
| d | | | 0.5053 (0.0154) | 0.9239 (0.0313) | | | 0.4572 (0.0869) | 0.9326 (0.1413) |
| γ | | -0.2879 (0.1141) | | -0.3189 (0.0886) | | -0.2866 (0.1231) | | -0.3114 (0.1372) |
| δ | | 1.4426 (0.1746) | | 1.3943 (0.2477) | | 1.2647 (0.2435) | | 1.2183 (0.3181) |
| ν | 4.6285 (0.3551) | 4.8183 (0.3685) | 4.5687 (0.2951) | 4.8390 (0.3411) | 4.7037 (0.3699) | 4.9348 (0.3860) | 4.6564 (0.3415) | 4.9708 (0.3811) |
| LL | -6 277 | -6 265 | -6 280 | -6 269 | -6 194 | -6 184 | -6 197 | -6 179 |
| BIC | 12 605 | 12 598 | 12 619 | 12 614 | 12 437 | 12 435 | 12 729 | 12 433 |

Table 5: Parameter estimates for Gold spot and futures returns between 01-Jan-2000 and 31-Dec-2016.

Firstly, spot prices of Gold and Silver feature an asymmetric variance response to shocks. Based on LL and Bayesian information criterion (BIC), the APARCH model outperforms long memory models and even the combined model FIAPARCH.

¹⁵A differentiation of *hedge*, *diversifier*, and *safe haven* is found in Baur & Lucey (2010, p. 219).

¹⁶Unfortunately, only front month futures prices for Platinum and Palladium are freely available but the source of the data could not be verified. In addition, these price series feature a very pronounced *inverse sawtooth* pattern in trading volume, which could bias prices to a certain extend. Hence, this data is excluded.

| | Silver spot ($n = 4427$) | | | | Silver SI3 ($n = 4268$) | | | |
|----------|----------------------------|---------------------|---------------------|---------------------|---------------------------|---------------------|---------------------|---------------------|
| | GARCH | APARCH | FIG | FIAP | GARCH | APARCH | FIG | FIAP |
| μ_0 | 0.0464 (0.0196) | 0.0506 (0.0203) | 0.0451 (0.0189) | 0.0497 (0.0145) | 0.0623 (0.0218) | 0.0634 (0.0200) | 0.0602 (0.0208) | 0.0582 (0.0582) |
| μ_1 | -0.0333 (0.0144) | -0.0403 (0.0157) | -0.0319 (0.0141) | -0.0373 (0.0145) | -0.0352 (0.0169) | -0.0399 (0.0185) | -0.0348 (0.0151) | -0.0384 (0.0339) |
| ω | 0.0110 (0.0043) | 0.0107 (0.0054) | 0.2846 (0.0718) | 0.0688 (0.1669) | 0.0113 (0.0042) | 0.0135 (0.0082) | 0.3244 (0.0961) | 0.1282 (0.0382) |
| α | 0.0366 (0.0051) | 0.0393 (0.0056) | | | 0.0345 (0.0054) | 0.0413 (0.0058) | | |
| β | 0.9603 (0.0049) | 0.9600 (0.0083) | 0.6296 (0.0654) | 0.6334 (0.1075) | 0.9651 (0.0044) | 0.9581 (0.0110) | 0.6185 (0.1502) | 0.6162 (0.1196) |
| ϕ | | | 0.3096 (0.0518) | 0.3147 (0.0585) | | | 0.3193 (0.0908) | 0.3260 (0.1069) |
| d | | | 0.3809 (0.0518) | 0.3706 (0.0977) | | | 0.3613 (0.0832) | 0.3479 (0.0731) |
| γ | | -0.2862 (0.0775) | | -0.2150 (0.0533) | | -0.2642 (0.1169) | | -0.1976 (0.0687) |
| δ | | 1.9122 (0.4546) | | 2.3238 (0.1960) | | 1.8488 (0.6956) | | 2.2925 (0.2613) |
| ν | 3.9193 (0.2511) | 4.1697 (0.2621) | 4.3783 (0.2668) | 4.1069 (0.2722) | 3.9630 (0.2532) | 4.3177 (0.2774) | 4.4585 (0.2836) | 4.2479 (0.3160) |
| LL | -8 337 | -8 312 | -8 326 | -8 315 | -8 248 | -8 225 | -8 241 | -8 231 |
| BIC | 16 724 | 16 692 | 16 712 | 16 706 | 16 546 | 16 519 | 16 540 | 16 538 |

Table 6: Parameter estimates for Silver spot and futures returns between 01-Jan-2000 and 31-Dec-2016.

The leverage parameter γ is negative and statistically significant. This indicates that positive shocks (price jumps) increase the variance at a larger scale than negative shocks. An illustration of this behavior is outline in Paper 5 (PM) during the turmoil of the ‘Brexit’ poll results.

Secondly, three months futures prices of Gold feature an increased long memory compared to its spot prices. Asymmetry is still present but at lower significance. FIAPARCH provides the best fit. For Silver, asymmetry has a decreased significance as well but the increased long memory is not as pronounced leaving APARCH as best model according to LL and BIC. We conclude that the econometric properties of spot and futures prices differ and address this phenomenon along with crude oil prices in the next subsection.

Thirdly, Platinum and Palladium are evidently different in their properties. The leverage parameter is very small and statistically indifferent from zero. Negative and positive returns have very similar impact on variance. For Palladium, the long memory models FIGARCH and FIAPARCH produce the same fit, yielding the lower BIC for FIGARCH. For Platinum, the LL of APARCH, FIGARCH, and FIAPARCH is similar and FIGARCH features the smallest BIC. We observe a very low $\delta = 1.3434$ and note that with this low Box-Cox transformation parameter the variance shifts from modeling squared returns to absolute returns which is an indication for an elevated degree of autocorrelation (Ding et al., 1993, pp. 96, 100).

Lastly, it is noted that these metal-specific properties are robust and parameter estimates are similar if the observation window is shifted. This is found when comparing the results to Paper 5 (PM) as well as to Hammoudeh & Yuan (2008, p. 619), Hammoudeh et al. (2010, pp. 638f.), Arouri et al. (2012, pp. 214f.), and Chkili et al.

| | Palladium spot ($n = 4354$) | | | | Platinum Spot ($n = 4429$) | | | |
|----------|-------------------------------|---------------------|--------------------|--------------------|------------------------------|---------------------|--------------------|---------------------|
| | GARCH | APARCH | FIG | FIAP | GARCH | APARCH | FIG | FIAP |
| μ_0 | 0.0388 (0.0202) | 0.0458 (0.0259) | 0.0478 (0.0245) | 0.0456 (0.0254) | 0.0409 (0.0177) | 0.0471 (0.0166) | 0.0456 (0.0166) | 0.0475 (0.0175) |
| μ_1 | 0.0499 (0.0167) | 0.0492 (0.0186) | 0.0490 (0.0164) | 0.0492 (0.0182) | 0.0156 (0.0141) | 0.0197 (0.0159) | 0.0216 (0.0159) | 0.0214 (0.0156) |
| ω | 0.0720 (0.0228) | 0.0559 (0.0210) | 0.3118 (0.0843) | 0.3822 (0.1340) | 0.0194 (0.0058) | 0.0211 (0.0080) | 0.1658 (0.0501) | 0.2630 (0.0549) |
| α | 0.0793 (0.0124) | 0.1116 (0.0272) | | | 0.0465 (0.0071) | 0.0705 (0.0115) | | |
| β | 0.9085 (0.0151) | 0.8884 (0.0189) | 0.5674 (0.1199) | 0.5760 (0.1888) | 0.9443 (0.0085) | 0.9294 (0.0129) | 0.5706 (0.1599) | 0.6112 (0.1971) |
| ϕ | | | 0.2790 (0.0728) | 0.2711 (0.0878) | | | 0.3133 (0.1020) | 0.2782 (0.0829) |
| d | | | 0.4421 (0.0818) | 0.4578 (0.1354) | | | 0.3734 (0.0854) | 0.4435 (0.1438) |
| γ | | -0.0289 (0.0992) | | 0.0234 (0.0426) | | -0.0397 (0.1419) | | -0.0213 (0.0616) |
| δ | | 2.2000 (0.2244) | | 1.8771 (0.2460) | | 1.3434 (0.1777) | | 1.6549 (0.2235) |
| ν | 5.5209 (0.4549) | 5.2528 (0.6038) | 6.2512 (0.5512) | 6.3391 (0.4738) | 6.0689 (0.5465) | 6.6138 (0.5878) | 6.6671 (0.6701) | 7.6867 (0.6924) |
| LL | -8 883 | -8 857 | -8 841 | -8 841 | -7 302 | -7 280 | -7 281 | -7 279 |
| BIC | 17 817 | 17 782 | 17 741 | 17 757 | 14 655 | 14 627 | 14 621 | 14 634 |

Table 7: Parameter estimates for Platinum and Palladium spot returns between 01-Jan-2000 and 31-Dec-2016.

(2014, pp. 9f.) only to name a few.

The intuition behind these models and their estimates could be explained in the following way. Gold is labeled safe-haven investment. As such, Gold should rise in price if a market is declining following the definition of Baur & Lucey (2010, p. 219). In the past, this price increase in Gold was jump-like and more violent the stronger the decline in a market was (negative shocks). This could cause overreaction as the demand for Gold increases tremendously, possibly yielding some minor price corrections thereafter. For Gold and Silver, this was observable in June 2016. Figure 6 plots the prices of Gold and Silver as well as squared returns and the *Two Scales Realized Volatility* proxy (Zhang et al., 2005) which is extensively discussed in Paper 5 (PM). The jumps in price and variance based on intra-day data are clearly visible, but declines relatively fast depicting short memory. For metals with industrial applications, the demand side should react slower to changes in the economy (cyclical slowdowns or upturns, crises, and booms). Production cycles cannot be changed immediately and storage is costly; hence, the demand is relatively price-inelastic (Yang, 2009, p. 1806f.). This also indicates that there should only be little if not zero asymmetry in returns.

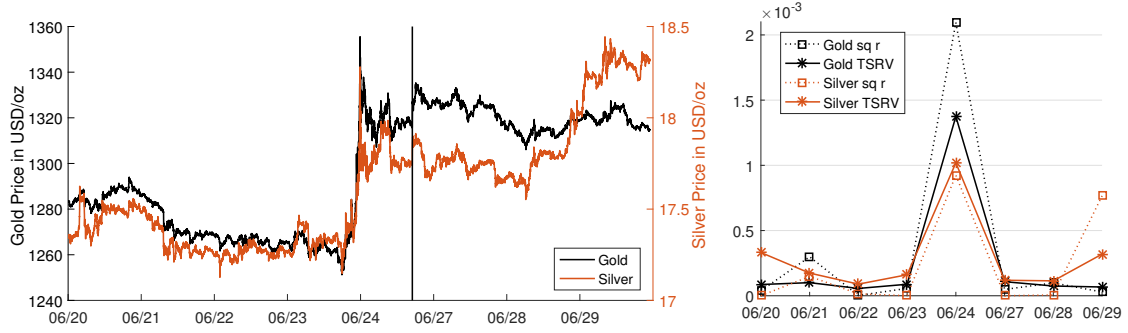


Figure 6: Gold and Silver prices (left) and variance measures (right) before and after the publication of the Brexit poll results on 24-Jun-2016. All times in *central day-light time* (CDT), data sampled from Bloomberg. The black vertical line marks the weekend. Source: Paper 5 (PM), p. 14.

3.2.2 Similarities to Crude Oil Returns and Concluding Remarks

In Paper 7 (Trends), spot and futures prices of maturity one, three, six, and twelve months of WTI and Brent crude oil are analyzed. The price data is sampled between 01-Jan-2007 and 29-Jul-2016 from Bloomberg as generic futures contracts CL m and COM m , respectively, where m denotes the maturity of the rolling contract. In order to assess the impact of short term or *micro* trends in returns on their variance, the asymmetric response is tested by applying variance models which explicitly depict this feature: APARCH and FIAPARCH. In Paper 7 (Trends), the estimates are used for inference of asymmetries and the effects of trend contagion on a market's variance.¹⁷

Here, we compare the estimates to the findings of the previous subsection; the differences in spot and futures prices in regard to varying asymmetry and long memory. Table 8 presents relevant parameter estimates and LL of spot and selected futures return series for GARCH, APARCH, and FIAPARCH (abbr. FIAP) with the assumption of Student's- t distributed errors. Most importantly, we observe an (statistically significant) asymmetry parameter of inverse sign compared to Gold and Silver. This holds true for all WTI and Brent series tested. A positive γ both in APARCH and FIAPARCH reveals that negative price movements increase the volatility on a larger scale than positive returns. This could be explained by the connection of crude oil and its prices with the economy, which is extensively and controversially discussed in literature (e.g. Hamilton, 1983, 2003; Kilian, 2009). If we compare the estimates for different maturities, we note that the asymmetric impact of news weakens with longer maturities. Long memory in variance is present, however, no general assertion on increasing shock persistence can be made from the parameter estimates as they are inconclusive. Additional long memory tests on the time series could answer this question in more detail. It is also noted that the sample size is considerably smaller compared to that of the previous subsection.

¹⁷See further: Paper 7 (Trends), pp. 21f.

| | | WTI | Brent | CL1 | CO1 | CL6 | CO6 | CL12 | CO12 |
|-------------|-----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| APARCH(1,1) | γ | 0.5539 | 0.4090 | 0.5107 | 0.3689 | 0.4165 | 0.3481 | 0.3356 | 0.2676 |
| | | (0.1464) | (0.1143) | (0.2309) | (0.1122) | (0.1295) | (0.1534) | (0.1161) | (0.0936) |
| | δ | 1.3069 | 1.7129 | 1.4207 | 1.5947 | 1.4729 | 1.6046 | 1.5798 | 1.6829 |
| | | (0.2649) | (0.3011) | (0.3701) | (0.1806) | (0.2778) | (0.4542) | (0.3577) | (0.2639) |
| | <i>LL</i> | 5 987.1 | 6 258.7 | 6 005.7 | 6 220.9 | 6 393.2 | 6 502.4 | 6 703.7 | 6 766.5 |
| FIAP(1,d,1) | γ | 0.2865 | 0.2409 | 0.2749 | 0.2287 | 0.2289 | 0.2264 | 0.2052 | 0.1919 |
| | | (0.1197) | (0.0987) | (0.0703) | (0.0871) | (0.0809) | (0.0712) | (0.0828) | (0.0921) |
| | δ | 1.7175 | 2.0677 | 1.8370 | 1.9693 | 1.9566 | 1.9624 | 1.9292 | 1.9664 |
| | | (0.3434) | (0.2917) | (0.2416) | (0.3872) | (0.1334) | (0.1316) | (0.1511) | (0.2535) |
| | d | 0.5134 | 0.4076 | 0.4615 | 0.4398 | 0.4305 | 0.4241 | 0.4526 | 0.4272 |
| | | (0.2271) | (0.1332) | (0.1404) | (0.1096) | (0.1293) | (0.1776) | (0.1843) | (0.1461) |
| | <i>LL</i> | 5 982.0 | 6 249.8 | 6 002.4 | 6 220.6 | 6 393.8 | 6 504.5 | 6 707.8 | 6 770.5 |
| GARCH(1,1) | <i>LL</i> | 5 973.2 | 6 245.9 | 5 991.6 | 6 213.2 | 6 383.9 | 6 495.3 | 6 697.8 | 6 762.2 |

Table 8: Asymmetric and long memory properties of WTI and Brent spot and selected futures prices from 01-Jan-2007 to 29-Jul-2016 ($n = 2\,421$). Log-likelihoods in bold font mark the rejection of the null hypothesis at 1% in a Likelihood-ratio test (null hypothesis: GARCH model). Source: Paper 7 (Trends), p. 22.

For both precious metals and crude oils, differences in variance properties of spot and futures returns are observed. Asymmetry is decreasing for both commodity classes. Long memory is increasing for Gold and Silver whereas the significance of these changes differs across assets. This extends the findings of Chkili (2016) who report parameters for spot and three months futures returns. The cause for these different characteristics might be the source of demand for spot and futures of different maturity. While spot markets deal with physical assets, futures markets are also speculatively driven. Contracts can be *rolled over* such that no physical involvement (delivery) or any other form of settlement becomes necessary. Futures are used for the three investment motives—*speculation*, *hedging*, and *arbitrage*—which further changes the demand side compared to spot markets. Cross hedging between oil and Gold futures markets is addressed in Narayan et al. (2010, pp. 3299f.) for example. Differences of spot and futures prices and their econometric properties of these metals is also connected to arbitrage opportunities and cost-of-carry (Blose, 2010; Theissen, 2012).

If these changes in characteristics hold for Platinum and Palladium as well could not be explored and is suited for further research. This could also involve important industry metals such as Aluminum, Copper, Zinc or fossil fuels like thermal coal or natural gas. Research on spot and futures markets, their characteristics, and participants is steadily increasing (e.g. Geman & Ohana, 2009; Fung et al., 2010; Silvennoinen & Thorp, 2013; Atil et al., 2014; Chkili, 2016).

4 Conclusions and Outlook

The essays presented as part of thesis are conceptualized to improve the understanding (and modeling, to that end) of the variance of returns of different asset classes. This includes asset-specific variance characteristics and their interpretation, application in financial risk management, and a perspective embedded in globalized markets with focus on spill-over and contagion between different markets. Therefore, the findings presented in this thesis are relevant to a broad audience of researchers, practitioners in risk and portfolio management, and to some extent, to policy makers.

Summarizing the most important findings for crude oil returns, a time-varying persistence structure in variance is revealed. Also, the price leadership of WTI towards Brent and the OPEC basket is endorsed as most short term trends are found to start in WTI prices and spill over to other markets. From the long-term perspective, markets share an underlying long term trend, supporting the ‘one great pool’ hypothesis originating from Weiner (1991). This has some interesting implications in view of the OPEC system and its desired influence on global crude oil prices—which cannot be confirmed for the last decade.

For precious metals, it is shown that including asset-specific variance characteristics improves variance predictions and VaR forecasts which is of relevance to portfolio management in terms of hedging. For this class of commodities, it is also found that squared daily returns are an insufficient measure for daily variance. Especially for Gold and Silver with a pronounced asymmetric response to market shocks, it is shown that using squared daily returns to measure forecasting quality yields biased results. This implies that squared daily returns as proxy in loss functions should be scrutinized. This is going to be addressed in future research as models for realized volatility are existent, e.g. the class of Heterogeneous Autoregressive Realized Volatility models (Corsi, 2009). However, given the introduction of FFT to variance modeling in Klein & Walther (2017), the classical GARCH-type models might be applied on extended data ranges such as intra-day data, too.

Since the applications in this thesis in terms of research objects are widespread, an even broader outlook on future research, motivated by results presented herein, can be outlined. Based on the findings for commodities and exchange rates, the variance and correlation models could be applied to stocks and stock indexes in order to determine if variance characteristics differ from other asset classes. Also, it would be of interest if different companies that are associated with identical sectors or industries have similar variance characteristics in their stock price returns. Possible asymmetries might cause some bias in pricing of derivatives with variance (or volatility) as a pricing factor. In the same vein, the influence of business cycles on stock-specific variance characteristics and robustness thereof could be addressed in future research.

In a multivariate setting, the reasons for volatility transmissions between assets and markets as well as the drivers of volatility are an appealing field of research, especially for commodity markets. At first, the author of this thesis will focus on direct extensions of the findings of this thesis such as an analysis and explanation of the temporal surrogate safe-haven behavior of Platinum. It is of interest if sequentially investing in Gold and Platinum suffice as a hedge and to what extent futures of different maturity improve potential hedging strategies. Changing asymmetric and long memory properties in futures returns should be further analyzed, tested, and explained as this has rarely been mentioned in literature so far. With the findings supporting the one great pool hypothesis for crude oil, more details on how short term contagion happens, its bi-directionality, and the causes for it are interesting questions to focus on in the field of energy finance and energy economics.

References

- Alonso, E., Field, F. R., & Kirchain, R. E. (2012). Platinum Availability for Future Automotive Technologies. *Environmental Science & Technology*, 46, 12986–12993. doi:10.1021/es301110e.
- Aroui, M. E. H., Hammoudeh, S., Lahiani, A., & Nguyen, D. K. (2012). Long memory and structural breaks in modeling the return and volatility dynamics of precious metals. *The Quarterly Review of Economics and Finance*, 52, 207–218. doi:10.1016/j.qref.2012.04.004.
- Atil, A., Lahiani, A., & Nguyen, D. K. (2014). Asymmetric and nonlinear pass-through of crude oil prices to gasoline and natural gas prices. *Energy Policy*, 65, 567–573. doi:10.1016/j.enpol.2013.09.064.
- Baillie, R. T., Bollerslev, T., & Mikkelsen, H. O. (1996). Fractionally integrated generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 74, 3–30. doi:10.1016/S0304-4076(95)01749-6.
- Baillie, R. T., & Chung, S.-K. (2002). Modeling and forecasting from trend-stationary long memory models with applications to climatology. *International Journal of Forecasting*, 18, 215–226. doi:10.1016/S0169-2070(01)00154-6.
- Batten, J. A., Ciner, C., & Lucey, B. M. (2010). The macroeconomic determinants of volatility in precious metals markets. *Resources Policy*, 35, 65–71. doi:10.1016/j.resourpol.2009.12.002.
- Batten, J. A., & Lucey, B. M. (2010). Volatility in the gold futures market. *Applied Economics Letters*, 17, 187–190. doi:10.1080/13504850701719991.
- Baur, D. G., & Lucey, B. M. (2010). Is Gold a Hedge or a Safe Haven? An Analysis of Stocks, Bonds and Gold. *Financial Review*, 45, 217–229. doi:10.1111/j.1540-6288.2010.00244.x.
- Blose, L. E. (2010). Gold prices, cost of carry, and expected inflation. *Journal of Economics and Business*, 62, 35–47. doi:10.1016/j.jeconbus.2009.07.001.
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31, 307–327. doi:10.1016/0304-4076(86)90063-1.
- Bollerslev, T., Engle, R., & Wooldridge, J. (1988). A capital asset pricing model with time-varying covariances. *The Journal of Political Economy*, 96, 116–131.

- Bollerslev, T., & Mikkelsen, H. O. (1996). Modeling and pricing long memory in stock market volatility. *Journal of Econometrics*, 73, 151–184. doi:10.1016/0304-4076(95)01736-4.
- Box, G. E. P., & Jenkins, G. M. (1976). *Time Series Analysis Forecasting and Control*. (2nd ed.). San Francisco: Holden-Day.
- Burnecki, K. (2012). *Identification, Validation and Prediction of Fractional Dynamical Systems*. Wroclaw: POIG.01.03.01-02-002/08.
- Caporin, M., & McAleer, M. (2013). Ten Things You Should Know about the Dynamic Conditional Correlation Representation. *Econometrics*, 1, 115–126. doi:10.3390/econometrics1010115.
- Celik, S. (2012). The more contagion effect on emerging markets: The evidence of DCC-GARCH model. *Economic Modelling*, 29, 1946–1959. doi:10.1016/j.econmod.2012.06.011.
- Chang, K.-L. (2012). Volatility regimes, asymmetric basis effects and forecasting performance: An empirical investigation of the WTI crude oil futures market. *Energy Economics*, 34, 294–306. doi:10.1016/j.eneco.2011.11.009.
- Chkili, W. (2016). Dynamic correlations and hedging effectiveness between gold and stock markets: Evidence for BRICS countries. *Research in International Business and Finance*, 38, 22–34. doi:10.1016/j.ribaf.2016.03.005.
- Chkili, W., Hammoudeh, S., & Nguyen, D. K. (2014). Volatility forecasting and risk management for commodity markets in the presence of asymmetry and long memory. *Energy Economics*, 41, 1–18. doi:10.1016/j.eneco.2013.10.011.
- Cont, R. (2001). Empirical properties of asset returns: stylized facts and statistical issues. *Quantitative Finance*, 1, 223–236. doi:10.1088/1469-7688/1/2/304.
- Cooley, J. W., & Tukey, J. W. (1965). An Algorithm for the Machine Calculation of Complex Fourier Series. *Mathematics of Computation*, 19, 297–301.
- Corsi, F. (2009). A simple approximate long-memory model of realized volatility. *Journal of Financial Econometrics*, 7, 174–196. doi:10.1093/jjfinec/nbp001.
- Davidson, J. (2004). Moment and Memory Properties of Linear Conditional Heteroscedasticity Models, and a New Model. *Journal of Business & Economic Statistics*, 22, 16–29. doi:10.1198/073500103288619359.
- Ding, Z., Granger, C. W. J., & Engle, R. F. (1993). A long memory property of stock market returns and a new model. *Journal of Empirical Finance*, 1, 83–106.

- Engle, R. (2002). Dynamic Conditional Correlation. *Journal of Business & Economic Statistics*, 20, 339–350. doi:10.1198/073500102288618487.
- Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, 50, 987–1007. doi:10.2307/1912773.
- Engle, R. F., & Kroner, K. F. (1995). Multivariate Simultaneous Generalized ARCH. *Econometric Theory*, 11, 122–150. doi:10.1017/S0266466600009063.
- Engle, R. F., & Ng, V. K. (1993). Measuring and Testing the Impact of News on Volatility. *Journal of Finance*, 48, 1749–1778. doi:10.2307/2329066.
- Fong, W. M., & See, K. H. (2002). A Markov switching model of the conditional volatility of crude oil futures prices. *Energy Economics*, 24, 71–95. doi:10.1016/S0140-9883(01)00087-1.
- Francq, C., & Zakoian, J.-M. (2004). Maximum likelihood estimation of pure GARCH and ARMA-GARCH processes. *Bernoulli*, 10, 605–637. doi:10.3150/bj/1093265632.
- Frigo, M., & Johnson, S. G. (2005). The design and implementation of FFTW 3. *Proceedings of the IEEE*, 93, 216–231. doi:10.1109/JPROC.2004.840301.
- Fung, H.-G., Wilson Liu, Q., & Tse, Y. (2010). The information flow and market efficiency between the U.S. and Chinese aluminum and copper futures markets. *Journal of Futures Markets*, 30, 1192–1209. doi:10.1002/fut.20474.
- Geman, H., & Ohana, S. (2009). Forward curves, scarcity and price volatility in oil and natural gas markets. *Energy Economics*, 31, 576–585. doi:10.1016/j.eneco.2009.01.014.
- Granger, C. W. J., & Joyeux, R. (1980). An Introduction to Long-Memory Time Series Models and Fractional Differencing. *Journal of Time Series Analysis*, 1, 15–29. doi:10.1111/j.1467-9892.1980.tb00297.x.
- Hamilton, J. D. (1983). Oil and the Macroeconomy since World War II. *Journal of Political Economy*, 91, 228–248. doi:10.1086/261140.
- Hamilton, J. D. (2003). What is an oil shock? *Journal of Econometrics*, 113, 363–398. doi:10.1016/S0304-4076(02)00207-5.
- Hammoudeh, S., Malik, F., & McAleer, M. (2011). Risk management of precious metals. *The Quarterly Review of Economics and Finance*, 51, 435–441. doi:10.1016/j.qref.2011.07.002.

- Hammoudeh, S., & Yuan, Y. (2008). Metal volatility in presence of oil and interest rate shocks. *Energy Economics*, *30*, 606–620. doi:10.1016/j.eneco.2007.09.004.
- Hammoudeh, S. M., Yuan, Y., McAleer, M., & Thompson, M. A. (2010). Precious metals-exchange rate volatility transmissions and hedging strategies. *International Review of Economics & Finance*, *19*, 633–647. doi:10.1016/j.iref.2010.02.003.
- Hansen, P. R. (2005). A Test for Superior Predictive Ability. *Journal of Business & Economic Statistics*, *23*, 365–380. doi:10.1198/073500105000000063.
- Hansen, P. R., Lunde, A., & Nason, J. M. (2011). The Model Confidence Set. *Econometrica*, *79*, 453–497. doi:10.3982/ECTA5771.
- Hosking, J. R. M. (1981). Fractional differencing. *Biometrika*, *68*, 165–176. doi:10.1093/biomet/68.1.165.
- Hurst, H. E. (1951). Long-term Storage Capacity of Reservoirs. *Transactions of the American Society of Civil Engineers*, *116*, 770–808. doi:10.1119/1.18629.
- Jensen, A. N., & Nielsen, M. O. (2014). A Fast Fractional Difference Algorithm. *Journal of Time Series Analysis*, *35*, 428–436. doi:10.1111/jtsa.12074.
- Johansen, S., & Nielsen, M. O. (2012). The role of initial values in nonstationary fractional time series models. *CREATES Working Paper*, *47*. URL: <http://econpapers.repec.org/RePEc:qed:wpaper:1300>.
- Kilian, L. (2009). Not All Oil Price Shocks Are Alike: Disentangling Demand and Supply Shocks in the Crude Oil Market. *American Economic Review*, *99*, 1053–1069. doi:10.1257/aer.99.3.1053.
- Klein, T. (2017). Dynamic correlation of precious metals and flight-to-quality in developed markets. *Finance Research Letters*, *23*, 283–290. doi:10.1016/j.fr1.2017.05.002.
- Klein, T., Pham Thu, H., & Walther, T. (2016). Evidence of Long Memory and Asymmetry in the EUR/PLN Exchange Rate Volatility. *Research Papers of Wrocław University of Economics*, *428*, 128–140. doi:10.15611/pn.2016.428.11.
- Klein, T., & Walther, T. (2016). Oil Price Volatility Forecast with Mixture Memory GARCH. *Energy Economics*, *58*, 46–58. doi:10.1016/j.eneco.2016.06.004.
- Klein, T., & Walther, T. (2017). Fast fractional differencing in modeling long memory of conditional variance for high-frequency data. *Finance Research Letters*, *22*, 274–279. doi:10.1016/j.fr1.2016.12.020.

- Landau, E. (1909). *Handbuch der Lehre von der Verteilung der Primzahlen*. Leipzig, Berlin: B. G. Teubner.
- Lawrance, A. J., & Kottegoda, N. T. (1977). Stochastic Modelling of Riverflow Time Series. *Journal of the Royal Statistical Society*, *140*, 1–47. doi:10.2307/2344516.
- Li, M., Li, W. K., & Li, G. (2013). On Mixture Memory GARCH Models. *Journal of Time Series Analysis*, *34*, 606–624. URL: <http://doi.wiley.com/10.1111/jtsa.12037>. doi:10.1111/jtsa.12037.
- Lintner, J. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics*, *47*, 13–37. doi:10.2307/1924119.
- Locarek-Junge, H., & Klein, T. (2016). Währungs-Hedging mit Derivaten. *WISU - Das Wirtschaftsstudium*, *45*, 298–304.
- Locarek-Junge, H., Klein, T., & Walther, T. (2014). GARCH-Modelle. *WISU - Das Wirtschaftsstudium*, *43*, 1348–1354.
- Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance*, *7*, 77–91. doi:10.2307/2975974.
- Massari, S., & Ruberti, M. (2013). Rare earth elements as critical raw materials: Focus on international markets and future strategies. *Resources Policy*, *38*, 36–43. doi:10.1016/j.resourpol.2012.07.001.
- McCulloch, J. H. (1985). Miscellanea: On Heteros*edasticity. *Econometrica*, *53*, 483.
- Mensi, W., Hammoudeh, S., & Kang, S. H. (2017). Dynamic linkages between developed and BRICS stock markets: Portfolio risk analysis. *Finance Research Letters*, *21*, 26–33. URL: <http://linkinghub.elsevier.com/retrieve/pii/S1544612316303245>. doi:10.1016/j.frl.2016.11.016.
- Mossin, J. (1966). Equilibrium in a capital asset market. *Econometrica*, *34*, 768–783. doi:10.2307/1910098.
- Narayan, P. K., Narayan, S., & Zheng, X. (2010). Gold and oil futures markets: Are markets efficient? *Applied Energy*, *87*, 3299–3303. doi:10.1016/j.apenergy.2010.03.020.
- Nomikos, N. K., & Pouliasis, P. K. (2011). Forecasting petroleum futures markets volatility: The role of regimes and market conditions. *Energy Economics*, *33*, 321–337. doi:10.1016/j.eneco.2010.11.013.

- Piontek, K. (2010). The analysis of power for some chosen VaR backtesting procedures: Simulation approach. In A. Fink, B. Lausen, W. Seidel, & A. Ultsch (Eds.), *Advances in Data Analysis, Data Handling and Business Intelligence* (pp. 481 – 490). Heidelberg: Springer-Verlag.
- Schatzman, J. C. (1996). Accuracy of the Discrete Fourier Transform and the Fast Fourier Transform. *SIAM Journal on Scientific Computing*, *17*, 1150–1166.
- Sensoy, A. (2013). Dynamic relationship between precious metals. *Resources Policy*, *38*, 504–511. doi:10.1016/j.resourpol.2013.08.004.
- Sharpe, W. F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance*, *19*, 425–442. URL: <http://www.jstor.org/stable/2977928?origin=crossref>. doi:10.2307/2977928.
- Silvennoinen, A., & Thorp, S. (2013). Financialization, crisis and commodity correlation dynamics. *Journal of International Financial Markets, Institutions and Money*, *24*, 42–65. doi:10.1016/j.intfin.2012.11.007.
- Sims, C. A. (1980). Macroeconomics and Reality. *Econometrica*, *48*, 1. doi:10.2307/1912017.
- Stoev, S., & Taqqu, M. S. (2004). Simulation Methods for Linear Fractional Stable Motion and FARIMA Using the Fast Fourier Transform. *Fractals*, *12*, 95–121. doi:10.1142/S0218348X04002379.
- Theissen, E. (2012). Price discovery in spot and futures markets: a reconsideration. *The European Journal of Finance*, *18*, 969–987. doi:10.1080/1351847X.2011.601643.
- Tse, Y. K. (1998). The conditional heteroscedasticity of the yen-dollar exchange rate. *Journal of Applied Econometrics*, *13*, 49–55. doi:10.1002/(SICI)1099-1255(199801/02)13:1<49::AID-JAE459>3.0.CO;2-0.
- Tse, Y. K., & Tsui, A. K. C. (2002). A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model With Time-Varying Correlations. *Journal of Business & Economic Statistics*, *20*, 351–362. doi:10.1198/073500102288618496.
- Vigne, S. A., Lucey, B. M., O'Connor, F. A., & Yarovaya, L. (2017). The financial economics of white precious metals - A survey. *International Review of Financial Analysis*, forthcoming, . doi:10.1016/j.irfa.2017.04.006.
- Walther, T., Klein, T., Thu, H. P., & Piontek, K. (2017). True or spurious long memory in European non-EMU currencies. *Research in International Business and Finance*, *40*, 217–230. doi:10.1016/j.ribaf.2017.01.003.

- Weiner, R. J. (1991). Is the world oil market ‘one great pool’?. *Energy Journal*, 12, 95–107. doi:10.5547/ISSN0195-6574-EJ-Vol12-No3-7.
- Yang, C.-J. (2009). An impending platinum crisis and its implications for the future of the automobile. *Energy Policy*, 37, 1805–1808. doi:10.1016/j.enpol.2009.01.019.
- Zhang, L., Mykland, P. a., & Aït-Sahalia, Y. (2005). A Tale of Two Time Scales. *Journal of the American Statistical Association*, 100, 1394–1411. doi:10.1198/016214505000000169.

Original Research Articles